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The present study describes how deaf and hard-of-hearing (d/hh) children who primarily use listening/spoken English (oral d/hh) solve one-step arithmetic story problems. Past research examined the story problem-solving of hearing children (Carpenter et al., 2015) and d/hh children who used age-appropriate American Sign Language (signing d/hh; Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). The present study, using descriptive and statistical analyses, examined overall findings regarding oral d/hh children's story problem-solving, compared findings between oral d/hh children with age-appropriate and below-age-appropriate spoken English comprehension, and considered results concerning past research from general and deaf education. Overall, the oral d/hh children used the same types of strategies as both hearing and signing d/hh children, but oral d/hh children's pattern of relative story problem difficulty was more similar to their signing d/hh peers than their hearing peers. Finally, this study found almost no significant differences between the one-step arithmetic story problem-solving of oral d/hh children with age-appropriate spoken English comprehension and with below-age-appropriate spoken English comprehension. The study concludes with limitations, implications for the classroom, and recommendations for future research.

THE ONE-STEP ARITHMETIC STORY PROBLEM-SOLVING OF DEAF/ HARD-OF-HEARING CHILDREN WHO PRIMARILY USE LISTENING AND SPOKEN ENGLISH

by

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APPROVAL PAGE

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CHAPTER I

INTRODUCTION

School Mathematics and Adult Outcomes

The need for mathematically-skilled employers has risen sharply over the past 18 years and is projected to increase. A report titled *STEM: Good Jobs Now and for the Future* stated that between 2000 and 2010, jobs in the fields of Science, Technology, Engineering, and Mathematics (STEM) grew three times faster than non-STEM jobs (Langdon et al., 2011). Furthermore, this report also predicted that from 2008 to 2018, the number of STEM-related jobs would grow at a faster rate (17%) than the number of non-STEM-related jobs (9.8%) (Langdon et al., 2011). A 2017 report updated Langdon et al.'s (2011) projection models, stating that STEM jobs grew by 14% while non-STEM jobs grew by only 1.7% from 2008 to 2017 (Noonan, 2017). While an analysis of Langdon et al.'s (2011) predictions for 2018 has yet to be published, Noonan's (2017) follow-up report still highlights a stark disparity in growth between STEM-related and non-STEM-related jobs. In sum, STEM-related jobs are expected to continue expanding more quickly than non-STEM jobs. Looking to the future, the U.S. government (Fayer et al., 2017; Noonan, 2017) and the Australian government (STEM Partnerships Forum, 2018) have both projected an increasing demand for STEM workers. For example, Noonan (2017) predicts STEM-related jobs will increase by 8.9% while non-STEM-jobs will increase by 6.4% through to 2024. Thus, the documented growth of employment opportunities within STEM fields since 2000, coupled with governments predicting continued growth in this sector, suggests STEM jobs and the employment opportunities they present are projected to increase.

Despite the rapid increase of jobs in STEM-related fields, data indicate that deaf/hard-of-hearing (d/hh) adults are entering STEM-related jobs at lower rates than their hearing peers (Garberoglio et al., 2017). Although the reason(s) d/hh adults enter STEM fields at lower rates than their hearing peers is/are unknown, mathematics skills are becoming increasingly important if d/hh adults are to increase their entry rate into, and successfully secure employment within, a changing job market.

Further studies suggest high school mathematics outcomes can impact not only adult employment outcomes, but also adult income levels, postsecondary graduation rates, and levels of physical health. Past studies indicate d/hh adults who graduate school with higher educational outcomes (including mathematics) tend to attain higher rates of employment on average (Hartmann, 2010; MacLeod-Gallinger, 1992; Schroedel & Geyer, 2000). Similarly, research linking mathematics outcomes and rates of employment found that hearing high school graduates with stronger mathematics skills tend to experience higher levels of full-time employment (Every Child a Chance Trust, 2009; Parsons & Bynner, 1997; Ritchie & Bates, 2013; Rivera-Batiz, 1992), lower levels of unemployment (Every Child a Chance Trust, 2009; Parsons & Bynner, 1997; Rivera-Batiz, 1992), and higher median income levels (Every Child a Chance Trust, 2009; Parsons & Bynner, 1997; Ritchie & Bates, 2013).

Other research has extended the benefits of high school mathematics outcomes beyond adult employment into other areas of adult life. Hearing high school students who enroll in advanced mathematics courses such as advanced algebra or calculus typically reap benefits including higher postsecondary graduation rates (Rose & Betts, 2001), higher overall career earnings (Black et al., 2015; Rose, 2006; Rose & Betts, 2001, 2004), and even improved physical health (Carroll et al., 2018).

Despite the documented importance of mathematics, research into d/hh learners' mathematics suggests the median mathematics scores attained by deaf and hard of hearing (d/hh) high school graduates on the Stanford Achievement Test, state assessments in Colorado and Arizona, and the Curriculum National Tests in Scotland, were all below grade level (Antia et al., 2009; Qi & Mitchell, 2011; Thoutenhoofd, 2006; Traxler, 2000). However, in all of these studies, the d/hh test-takers took tests normed on hearing test-takers (Antia et al., 2011; Thoutenhoofd, 2006; Traxler, 2000), though adaptations such as written instructions, extra time, and a screening placement test were utilized some of the time (Qi & Mitchell, 2011). Assessment researchers suggest when one group of people are assessed using a test normed on another group of people that differ culturally, linguistically, or experientially, results can be biased and possibly misrepresent the test-takers' true skills and abilities (Kim & Zabelina, 2015; Kim et al., 2016; Miller, 2015). Thus, these tests may have been biased against d/hh test-takers, artificially lowering their mathematics scores.

In addition, the authors used (Antia et al., 2011; Thoutenhoofd, 2006; Traxler, 2000) used median scores instead of mean scores within a standard deviation to support their conclusions. More recent research has shown that mean-based statistical analyses can yield very different conclusions about mathematics performance of d/hh students (Henner et al., under review). In their analysis of d/hh student performance on the MAP Mathematics subtest, Henner and his colleagues (under review) compared the overlap of score standard deviations instead of score means and concluded that many of the d/hh participants' scores reflected a performance far closer to their hearing peers than median comparisons would suggest.

Nevertheless, there is a suggestion that many d/hh learners traditionally do not graduate high school as mathematically prepared as their hearing peers. This suggestion highlights the importance of monitoring and, where necessary, increasing the mathematics outcomes of *all* high school graduates, including those who are d/hh. Ensuring d/hh students graduate high school mathematically skilled is critical given the increasingly important role mathematics outcomes play in employment outcomes and postsecondary outcomes.

Research suggests raising high school mathematics outcomes begins in the primary grades. Three studies have explicitly shown that mathematics scores collected in primary grades (i.e., K–3) can statistically predict mathematics scores in the upper grades (i.e., Grades 5–10). The first study included 196 hearing children in the United States (Jordan et al., 2009), the second included one dataset of 3,677 hearing children born in the United Kingdom, and the other included 599 hearing children born in the United States (Siegler et al., 2012). From these results, supporting and raising mathematics outcomes in the lower grades can have long-term benefits.

Story Problem-Solving as a Foundational Mathematics Skill

Story problem-solving plays a unique role in a child's mathematics education. Mathematics comprises many skills (e.g., counting, problem-solving), knowledges (e.g., number sense, patterning), and disciplines (e.g., algebra, geometry, discrete mathematics). Of these, it has been suggested story problem-solving plays a unique role in developing one's understanding of the world. In his book *The Joy of X: A Guided Tour of Math from One to Infinity*, Stanford statistician Steven Strogratz (2012) describes the importance of story (word) problems as follows:

> Word problems give us practice in thinking not just about numbers, but about the *relationships* between numbers . . . they express the inner logic of the world around us. Cause and effect, supply and demand, input and output, dose and response—all involve pairs of numbers and the relationships between them. (p. 65)

This quote suggests story problems play a unique role in people's lives because they provide an opportunity to consider quantities and how they are connected and expressed through language. Such connections can also be expressed through mathematical notation, for example algorithms (i.e., 2 + 3 = ?), equations (i.e., 2x x 3 = 30), ratios (i.e., 2:4), and coordinate pairs (i.e., (3, 9)). This distinction illustrates story problems as providing a unique, language-based avenue into mathematics, specifically the relationships between quantities.

Studies highlight the importance of problem-solving experiences, including story problem-solving, within children's mathematics development. A literature summary of worldwide research into STEM engagement suggests allowing children in the early grades to wrestle with problems, including story problems, can lead to “deep mathematics learning” (Tytler et al., 2008, p. 44)—understanding why and how mathematics works. Given that the experience of solving mathematics problems provides children unique opportunities to think about the world and to come to understand mathematics on a deeper level, one can reason that mathematics story problems are critical learning experiences in a child’s education.

Further supporting the importance of story problem-solving in education, three of the leading standards in mathematics education in the United States—set by the National Council for Teachers of Mathematics (Martin, 2000), the Common Core State Standards (CCS, 2020), and the National Research Council (NRC, 2001)—each includes one strand devoted solely to problem-solving (including story problem-solving) for kindergarten to Grade 8 mathematics proficiency. These three documents, representing national U.S. policy and national U.S. educational standards, describe what mathematically-proficient children should be able to in their first 8 years of formal schooling. Defining problem-solving skills (including story problem-solving) as a standalone topic across all three documents reinforces the foundational importance of story problem-solving in a child’s mathematical education. Given the importance of story problem-solving in mathematics development, research supporting d/hh children’s story problem-solving development is critical.

Limitations of Story Problem-Solving Research Within Deaf Education

Research of d/hh children’s story problem-solving is limited in several ways. Much of the research is at least 10 years old, and the field lacks research on story problem-solving in general. For example, a preliminary literature search from 2010 to 2020 in Academic Search Complete found 296 papers published under the Boolean phrase "word problem solving" and only four papers published under the Boolean phrase "word problem solving AND deaf" in that same period.

While indeed not an exhaustive search, the ratio of 296 general education papers to four deaf education papers over a decade starkly indicates the relatively low volume of recent research regarding d/hh children's story problem-solving. Another concerning factor of this research base is the use of communicatively heterogeneous samples; that is, samples that include some combination of d/hh children who use sign language, d/hh children who use listening/spoken language, and d/hh children who use a communication system (i.e., Sign Supported Speech, Cued Speech).

Of the 19 known deaf education studies examining story problem-solving with d/hh children/youth, nine included a communicatively heterogeneous sample, and none separated findings by participant language/communication system used to address this lack of sampling clarity. The remaining 10 studies included homogeneous samples; of these, nine included d/hh children who used a signed language (ASL and Libras), and one included d/hh children who used listening/spoken Spanish.

As story problems are accessed, comprehended, and solved using language, studies linking findings back to one specific language present information that is entirely applicable to one group of d/hh children based on their language use. In other words, studies with communicatively heterogeneous samples that do not attribute specific findings to specific participant groups potentially report results that do not reflect all subgroups within the population. Such results lack clarity, in turn diminishing the research-based support professionals can offer d/hh children as they develop story problem-solving skills in the critical primary years.

Multiple researchers have discussed concerns regarding sample heterogeneity within deaf education research. Powers et al. (1998) stated the heterogeneity of this population was difficult for researchers to overcome when investigating and reporting children's educational achievements. Swanwick and Marschark (2010) elaborated on this, stating, "the range of educationally-relevant individual differences in this [d/hh] population is larger than it would be in a similarly sized population of hearing children" (p. 221). In other words, the heterogeneity of children within deaf education presents a challenge that is not necessarily present within general education research. Researchers, to address this challenge, have suggested that the generalization of research findings in deaf education is not possible on the same scale as it is in general education and that researchers and research consumers need to recognize this limitation when reporting and using findings (Marschark & Spencer, 2010a; Marschark et al., 2011). These ideas show that sample heterogeneity in deaf education has been a concern for some time, and suggest that one response to this concern is to accept sample heterogeneity as a limitation of research.

However, the variables that create sample heterogeneity in deaf education (i.e., languages/communication modes, educational settings, hearing levels, assistive listening device use, etc.) *can* be accounted for if research is designed to limit findings to specific variables. While accounting for this heterogeneity leads toward small sample sizes and away from the ‘gold standard’ of large-scale randomized trials, it also leads to targeted findings that teachers and researchers can confidently apply to the correct subgroup(s). In summary, heterogeneous sample makeup is concerning because it leads to knowledge regarding d/hh children that lacks clarity and applicability.

In response to these concerns regarding the story problem-solving literature in deaf education, applying studies from general education to deaf education might seem an appropriate response because they include relatively more homogeneous samples, and there are more from which to draw. Indeed, multiple researchers in deaf education have suggested that this practice, while not ideal, is necessary (Marschark & Spencer, 2010b; Marschark et al., 2011). However, mathematics development research cautions against such a cross-population application of research findings.

For example, a statistical analysis of factors predicting German d/hh children’s (N = 24) and hearing children’s (N = 24) arithmetic computation test scores in second through fourth grade found the children’s arithmetic scores did not significantly differ between groups; however, d/hh children’s arithmetic scores were positively related to reading comprehension but not nonverbal IQ, while hearing children’s arithmetic scores were positively associated with nonverbal IQ but not reading comprehension (Huber et al., 2014). Had this study not been conducted, the possibility that hearing and d/hh children’s arithmetic computations are not impacted in identical ways though they are identical in outcome would not have come to light.

In another study comparing 3- and 4-year-old d/hh (N = 10) and hearing (N = 10) preschoolers’ correctness when reproducing numbers, Zarfaty et al. (2004) found correctness rates differed depending on whether the number was represented in a spatial or temporal context. In both contexts, children were shown a collection of blocks on a computer screen and then asked to re-create the image from memory using real-life blocks. In the spatial condition, the blocks appeared on the screen as a single image; in the temporal condition, the blocks appeared on the screen one at a time. D/hh children and hearing children were equally accurate in reproducing temporally represented numbers, while d/hh children were more accurate than hearing children in producing spatially represented numbers. From this study emerged the indication that d/hh and hearing preschoolers recreate numbers with varying degrees of correctness depending upon the context.

These two examples indicate what can happen when research findings from hearing children are assumed to reflect d/hh children as well: Teachers, researchers, and other stakeholders may have acted upon assumptions that d/hh and hearing children develop arithmetic computation and number representation skills within identical trajectories. As Pagliaro (2015) suggests, setting expectations and making pedagogical decisions within a mathematics development trajectory that is inappropriate for a d/hh learner can lead to surface-level, procedural learning instead of deep understanding, further limiting that learner's mathematics performance through the grades. Only because of studies like these (Huber et al., 2014; Zarfaty et al., 2004), that include d/hh children, do we know when learning expectations and trajectories cannot be applied directly from general to deaf education. Thus, the most powerful way to understand d/hh children's development is to work with d/hh children. Extending this conclusion to story problem-solving, the most powerful way to understand d/hh children's story problem-solving is by working with them on story problem-solving tasks.

The Listening/Spoken Language Students Within Deaf Education

While there is a need for further research into all d/hh children's story problem-solving, the number of d/hh children who primarily use listening/spoken language highlights a particular need for further communicatively heterogeneous research with this population/subgroup. Statistics regarding the number of U.S. d/hh children who received instruction, at least partially, through spoken language were collected from 1968 to 2014 by the Gallaudet University Research Institute through the *Annual Survey of Deaf and Hard of Hearing Children & Youth* (Gallaudet Research Institute, 2014). This survey collected audiological, demographic, and educational information about children with hearing loss in kindergarten to Grade 12. The data were reported primarily by teachers. Data for the years 2000 to 2014 are publicly available (2008-2009 and 2012-2013 are unavailable).

From the years 2000 to 2004, the percentage of survey respondents reporting d/hh children receiving at least some instruction in spoken language (Gallaudet Research Institute, 2000, 2001, 2002, 2003, 2004). Then, from the years 2005 to 2014 (sans 2008-2009 and 2012-2013), more than 50% of children were reported to receive at least some instruction in spoken language each year, with percentages ranging from 51.3% to 53% (Gallaudet Research Institute, 2005, 2006, 2007, 2009, 2010, 2011, 2013, 2014). These percentages suggest that for the first 14 years of the millennium, a significant portion of the d/hh student population, if not the majority, received their education via spoken language, at least partially. No known comparable data have been collected from 2014 to 2020. Within this 4-year interim, it is possible that oral d/hh students have come to no longer represent such a large proportion of the d/hh population, but such a shift would require the reversal of a consistent 14-year trend.

More recently, data indicating the prevalence of d/hh children spending at least part of their school day in a general education environment has been collected and published by the National Center for Education Statistics. Summary reports, collected each year from 2015 to 2018, described the percentages of d/hh students, ages 6 through 21, receiving instruction in a variety of educational environments including public (general education) schools, schools for the deaf, homeschooling, and hospital/ homebound. Findings indicated 86% to 88% of respondents received some instruction in a general education classroom (Snyder et al., 2016, 2018), and 61%-63% spent most of their day there, where spoken English is presumed to be the language of instruction. This report does not indicate how many of these d/hh children actually accessed the content through spoken English and it is possible that some of them attended general education classrooms while receiving instruction through an ASL interpreter or language facilitator. As a note, these data do not necessarily indicate that the general education setting was the most appropriate placement, only that the child was in the setting. However, d/hh children who use listening/spoken language likely continued to be the majority from 2015 to 2018 given the relatively high percentages of children attending these settings: 80% attended at least some of the time each day, and over 60% spent the majority of their day there. Looking across results from the *Annual Survey of Deaf and Hard of Hearing*

Children & Youth (Gallaudet Research Institute, 2014) and NCES (Snyder et al., 2016, 2018), listening and spoken language has been a critical factor in the education of d/hh children for almost 2 decades.

Given that STEM jobs are increasing but d/hh entry into this field is not keeping pace, that mathematics outcomes can lead to higher postsecondary outcomes and story problems are a unique and critical topic within mathematics education, that d/hh children's mathematics is best understood by working with d/hh children, and that potentially the majority of d/hh children in general education settings use listening/spoken language, research into oral d/hh children's story problem-solving is needed. This statement is not meant to preclude the continued need for research with children who use sign language or children who use communication systems; rather, it is intended to reinforce the need for the present study.

Definition of Terms

Assistive Listening Device (ALD)—Technology that gives the user access to acoustic (sound) information. There are different types of devices available to d/hh children, including hearing aids, cochlear implants (CIs), bone-anchored hearing aids (BAHAs), and personal FM systems.

Bone-Anchored Hearing Aid (BAHA)—A device worn on the skull behind the ear. It takes sound waves from the environment and transmits them to the brain through vibrations in the skull.

Cochlear Implant (CI)—A device consisting of two parts: an internal part surgically implanted under the scalp, and an external part worn behind the ear, much like a hearing aid. Together, they take sound waves from the environment and transmit them to the brain through the external and internal parts by transforming the sound waves into electro-acoustical energy.

Cognitively Guided Instruction (CGI)—A professional development program wherein mathematics teachers are provided information about research on children's story problem-solving and conceptual frameworks to help them think about their own students' story problem-solving.

Communication System (CS)—An artificial system that combines spoken language with signs or visual cues. Examples include Cued Speech, Sign Supported Speech, and Signing Exact English.

Deaf/Hard of Hearing (d/hh)—the term used to describe a person, child, or adult having an audiological measure of hearing loss, ranging from "mild" to "profound."

FM (Frequency Modulated) System—Personal relay system in which a transmitter picks up sound from the environment and sends it to a receiver connected to the child's assistive listening device, giving the child direct access to sound without background noise.

Hearing—the term used to describe a person, child, or adult having an audiological measure within the normal range for children (-10 to 15 Hz; Bess & Humes, 2003) and having no known disabilities.

Hearing Aid—A device worn inside or outside the ear that takes in and amplifies sound waves from the environment for the inner ear to transform into electro-acoustical energy and send it to the brain.

One-Step Arithmetic Story Problem-Solving—A description of a story situation that identifies a relationship between two known quantities, with the objective of finding a third. For example, Lily has five flowers. She gave two to her mother. How many flowers does Lily have now?

CHAPTER II

REVIEW OF THE LITERATURE

Introduction

Adults with higher mathematics outcomes tend to experience lower rates of unemployment, higher rates of full-time employment, and higher median income levels (Parsons & Bynner, 1997; Ritchie & Bates, 2013; Rivera-Batiz, 1992). This indicates high school graduates' mathematics outcomes (i.e., knowledge and skills) are important for securing opportunities in adult life. However, research suggests d/hh children's median scores of mathematics computation and problem solving are below grade level (e.g., Gottardis et al., 2011; Qi & Mitchell, 2011; Traxler, 2000). Using research from the fields of general and special education, this chapter describes the mathematics outcomes of hearing children and d/hh children, and shows how research into story problem-solving for d/hh children who primarily use listening/spoken English can help the field of deaf education understand, and thus target, these median below-grade-level mathematics outcomes.

It is important to recognize that there are many factors that correlate with hearing children's mathematics outcomes; examples of these factors include age (Grissom, 2013; Oshima & Domaleski, 2006), socioeconomic status (Anders et al., 2012; Denton & West, 2002), and teacher knowledge of mathematics content and pedagogy (Hill et al., 2005; Jürgen et al., 2010). These factors imply no hearing child is truly "typical." However, to broadly compare mathematics outcomes across hearing and d/hh populations, discussions of hearing children's mathematics research will not distinguish between these factors.

One way the field of deaf education describes its children is by language and/or communication system use. Languages can be signed, spoken, or written; spoken language can be provided either through speech only or speech supplemented with signs or with cues (i.e., an artificial communication system). Using these descriptors, d/hh children can be described as "those who use a signed language," "those who use listening/spoken language," or "those who use a communication system." Given these categories, four phrases will describe subpopulations represented in the research: (a) "d/hh children," indicating either all or some combination of d/hh children who use a signed language, a communication system, and/or a listening/spoken language to communicate, (b) "signing d/hh children," indicating only participants who primarily use American Sign Language (ASL) to communicate, (c) "oral d/hh," indicating only participants who primarily use a listening/spoken language to communicate, and (d) "CS d/hh children," indicating only participants who use a communication system.

This literature review first discusses what is known and unknown about mathematics achievement in general and with d/hh students. From this, story problem-solving, as a specific mathematics skill, is defined and what is known and unknown about factors impacting story problem-solving within general and deaf education are presented. The conceptual frameworks built to analyze children's story problem-solving (Carpenter et al., 2015) are defined, and research using these frameworks to describe oral d/hh story problem-solving and hearing children's story problem-solving is summarized. This leads into the purpose of the study and the study's theoretical framework, and concludes with the research questions.

Mathematics Achievement

General Education

Recent mathematics scores on the National Assessment of Educational Progress (NAEP), a test given to fourth- and eighth-graders across the United States, highlight the need for specific research into d/hh children's mathematics. Large-scale testing is not specific enough to indicate d/hh children's performance in comparison with the hearing test-taking majority. The most recent NAEP was conducted in 2017, where the average mathematics scores were significantly ($p < .05$) higher than scores in the late 1990s and early 2000s (NAEP, 2017). This suggests children's mathematics scores have improved over the years. However, d/hh children bring additional variables to their learning that hearing children do not, including Deaf cultural considerations, differences in executive functioning and memory, and/or possible language and academic delays stemming from language deprivation and a lack of complete access to language (Marschark et al., 2011). D/HH children's mathematics development can also be impacted by their cognitive organization—because they access the world differently than hearing children—parental engagement and language use, background knowledge, and teacher preparation in mathematics (Pagliaro, 2015). As Marschark et al. (2011) state, "the deaf learner should not be viewed as a hearing learner who cannot hear" (p. 220). In other words, it cannot be assumed that practices and understanding grounded in general education generalize well to d/hh students. Therefore, while recent NAEP (2017) scores suggest average mathematics scores are improving, where d/hh children's NAEP scores fall in relation to this average is unknown because the scores of d/hh students were not disaggregated from the overall findings. Truly understanding d/hh children's mathematics outcomes requires research with d/hh children.

Deaf Education

Studies suggest many d/hh children's mathematics outcomes are below grade/age level at preschool, elementary, and high school levels (e.g., Edwards et al., 2013; Qi & Mitchell, 2011; Sarant et al., 2015; Traxler, 2000). National scale standardized assessments show a noted grade-level gap between the scores of hearing high school graduates and d/hh high school graduates on tests of mathematics skill and mathematics problem solving (Qi & Mitchell, 2011; Traxler, 2000). On a version of the Stanford Achievement Test (SAT) ninth edition, d/hh high school graduates' median problem-solving scores were equivalent to the 5.8 grade level (Traxler, 2000). Building upon this, Qi and Mitchell (2011) analyzed scores of d/hh students who took the Stanford Achievement Test for the Hearing Impaired (SAT-HI: Trybus & Karchmer, 1977). The SAT-HI is a version of the SAT that includes three accommodations: a screening test, practice test items, and printed instructions (as opposed to oral instructions). These accommodations were an effort to make the test more accessible and therefore, as Qi and Mitchell (2011) claimed, more reliable and valid for d/hh test-takers. Results showed d/hh high school graduate's median scores in mathematical problem-solving ranged between Grade 3.8 and Grade 5.7 equivalency levels (Qi & Mitchell, 2011). Recent research, however, indicates a different conclusion than Traxler (2000) and Qi and

Mitchell (2011). By comparing the overlap of standard deviations, instead of comparing across medians or means, as past studies have done, Henner et al. (in press) concluded many of the d/hh students demonstrated mathematical scores relative to their hearing peers. To be clear, some of the d/hh students still scored below expected levels on the MAP mathematics subtest, but the overall picture of d/hh student's mathematical ability is far less bleak (Henner et al., under review). Thus, the documented 46-year range from 1974 to 2020 shows below-grade-level problem-solving scores have been, and continue to be, a concern within deaf education.

Another longitudinal study using state-wide assessment scores of d/hh children in integrated settings (classes with hearing peers) in Colorado and Arizona showed slightly different findings than Traxler (2000), Qi and Mitchell (2011), and Henner et al. (in press). Participants in this study were integrated with hearing peers for 2 or more hours per day. Antia and her colleagues (Antia et al., 2009) followed their academic progress beginning in Grades 2–8, following them for 5 years through to Grade 7 to high school graduation. Over these 5 years, 63% to 79% of test-takers (*N* = 197) scored average or above average on the mathematics test (Antia et al., 2009). Of the 29-37% who scored below average, their gap remained stable over 5 years of data collection. These percentages indicate that while d/hh children educated in general education settings can develop mathematically at a similar pace to their hearing peers, more research into their mathematics development is urgently needed if the documented gap is to close and not simply remain stable.

Still, the study was neither specific enough in scope to document patterns in mathematics outcomes across participant variables (i.e., grade level or time spent in mainstream settings) nor specific mathematics skills (i.e., story problem-solving). Thus, there is no understanding about what led many of the d/hh test-takers to develop on par with their hearing peers while a significant portion demonstrated a stable gap, with mathematics scores below grade level each of the 5 years. Studies with more specific designs can inform this current lack of knowledge.

Two other studies accounted for general mathematics outcomes (Kritzer, 2009; Pagliaro & Kritzer, 2013), and by including only preschool participants, were more focused in age range than Antia et al.'s (2009) study. This smaller sample age range allowed findings to be attributed directly to preschool d/hh children, expanding the field's knowledge of d/hh children's mathematical outcomes beyond the school-age findings of Antia et al. (2009), Traxler (2000), Qi and Mitchell (2011), and Henner et al. (in press).

Results from both Kritzer (2009) and Pagliaro and Kritzer (2013) indicate a delay in mathematics scores, relative to the hearing norming sample, on the Test of Early Mathematical Ability-3 (TEMA-3; Ginsburg & Baroody, 2003) (Kritzer, 2009; Pagliaro & Kritzer, 2013). The TEMA-3 is a standardized assessment, normed on hearing children without disabilities, and measures a young child's foundational concepts of number. One mathematics task both Kritzer (2009) and Pagliaro and Kritzer (2013) noted as being particularly difficult for the d/hh preschool participants was story problem-solving. While such difficulty cannot be called a "delay"—as many hearing children are not exposed to story problem-solving before kindergarten, and thus hearing children would possibly struggle with this topic as well—such difficulty does show the potential for young d/hh children to struggle with story problem-solving relative to other areas of mathematics development. Given the smaller sample sizes of these studies—N = 28 (Kritzer, 2009) and N = 20 (Pagliaro & Kritzer, 2013)—it is also possible that these findings reflect that the TEMA-3 (Ginsburg & Baroody, 2003) story problems were out of the developmental range for *those* d/hh children, but perhaps not the majority of d/hh preschool children. Further research examining story problem-solving more closely can help clarify these possibilities.

This body of research regarding the overall mathematics outcomes of d/hh children has two noted limitations. First, all these studies used measures standardized on hearing children to evaluate d/hh children's mathematics proficiencies. This is an issue, as assessments designed for a specific population but then used to measure a different population can result in biased or erroneous scores. In other words, findings drawn from such assessments may not truly reflect the non-standardized (e.g., d/hh) population's mathematics outcomes. Secondly, all findings were drawn from studies that are known or assumed to have included children of different linguistic backgrounds (i.e., signing d/hh, oral d/hh, and/or CS d/hh children). Additionally, four of the six studies (Antia et al., 2009; Henner et al., under review; Qi & Mitchell, 2011; Traxler, 2000) combined different educational settings (i.e., general education settings, specialized programs housed in a public school, and/or schools for the deaf). This lack of distinction between variables means findings cannot be attributed to specific languages or communication modalities, specific mathematical skills, and/or specific educational settings. Due to their national scale, these studies inform a general understanding of d/hh children's mathematics outcomes, creating a useful "birds-eye view" of d/hh children's mathematical strengths and continued areas of development for the field of deaf education. However, researchers, teachers, and professionals cannot use such broad findings to directly inform their work with specific children in specific educational settings who use specific language(s) and/or communication systems.

Additionally, these six studies measured a wide scope of mathematics concepts and skills (i.e., story problem-solving, arithmetic, counting, number sense, geometry, etc.) and did not report scores of specific concepts and skills. Researchers in the field of deaf education have stated studies separated into specific mathematics knowledges and skills would be more explicit, and therefore usable, for supporting specific areas of mathematics (Sarant et al., 2015). Story problem-solving is unique within mathematics education because it represents quantities, and how they are related, solely through writing, speaking, and/or signing without the use of mathematical notation (i.e., +, ÷, ≥). However, it is unclear how well the conclusion reached by these studies—that some d/hh children are below age/grade level in their mathematics—represents actual performance on story problem-solving tasks.

Although d/hh children's median below-age-/grade-level performance in mathematics has been identified from preschool to postsecondary levels, explicit research aimed at targeting these broad findings should begin at the primary level with story problem-solving. Beginning at the primary level is important because what occurs in the younger grades impacts the upper grades. Longitudinal studies of literacy in deaf education show that outcomes at younger ages (Grade 3 and lower) can predict outcomes two, three, and four grades higher (Kyle & Harris, 2010, 2011; Spencer & Oleson, 2008). Similarly, longitudinal studies of mathematics in general education show children's outcomes in primary grades are positively correlated with their outcomes in upper elementary and high school grades (Jordan et al., 2009; Siegler et al., 2012). These findings suggest that one way to address the below-grade-level mathematics performance of d/hh, high school graduates (e.g., Antia et al., 2009; Henner et al., under review; Qi & Mitchell, 2011; Traxler, 2000) is to examine mathematics at younger grade levels. Due to the scale of this research, however, specific knowledge regarding primary-aged d/hh students' story problem-solving cannot be developed.

Problem-Solving

The Importance of Story Problem-Solving

Supporting d/hh children's story problem-solving outcomes is important because this skill is foundational not only to school outcomes, but to adult outcomes as well. In adult life, mathematically understanding and representing situations—that is, understanding how quantities relate to one another and manipulating those relationships to solve problems—are important (Strogratz, 2012). Adults problem-solve when they balance their bank account, write a fiscal report, shop with a budget, or plan a road trip. Solving story problems in primary school is considered a foundational form of this mathematical skill, the structured version one practices with before moving on to more complex, less-defined, real-life problems (Carpenter, 1985; Carpenter et al., 1993; Jonassen, 2000). However, multiple researchers suggest that many d/hh children and adults do not attain these story problem-solving skills at preschool, elementary, high school, or postsecondary levels (e.g., Blatto-Vallee et al., 2007; Gottardis et al., 2003; Kelly & Mousley, 2001; Kritzer, 2009; Pagliaro & Kritzer, 2013; Qi & Mitchell, 2011; Traxler, 2000). Thus, increasing d/hh children's story problem-solving understanding is critical not only as a set mathematical skill but also as a means to adult personal and professional ends, given that mathematics outcomes have been linked to postsecondary graduation rates, securing full-time employment, and income levels, as previously described.

Defining Story Problem-Solving

Martinez (1998) suggests that people have engaged in problem-solving when they "achieve something without having known beforehand how to do so . . . [making] your way toward your goal step by step, making some false moves but gradually moving closer toward the intend ed end point" (pp. 605–606). In other words, problem solving occurs when the path or process from point A to point B is not known beforehand *and* when people find their way to point B through reflection, reasoning, and critical thinking. This phrase "problem-solving" is broad and can include anything from using physics to construct a new type of roller coaster, to devising a new knitting pattern, to finding a coffee shop in an unfamiliar town. *Story* problems express a relationship between two or more quantities (numbers) couched within a short story (Lesh & Zawojewski, 2007; Strogratz, 2012). Thus, story problem-solving is the process of finding an unknown quantity expressed in a story, where the path to identifying that unknown quantity is not known beforehand. According to the *Common Core State Standards* (2020), the process of solving problems requires children to first explain (to themselves) the situation within a problem, in order to understand what solutions might be possible, and finally devise and execute a pathway from point A to B (CCSSO/NGA, 2020). One specific type of story problem is the one-step arithmetic story problem.

The *Merriam-Webster Dictionary* defines arithmetic as follows: “usually the nonnegative real numbers . . . [and] applying operations of addition, subtraction, multiplication and division to them” (Arithmetic, n.d.). Given that this study is contained within kindergarten to Grade 3 and negative numbers and rational numbers are generally not a core part of the curriculum until after third grade (CCSSO/NGA, 2020), the term “arithmetic” here is restricted to the operations of addition, subtraction, multiplication, and division performed on whole-number quantities. One-step arithmetic story problems, therefore, require children to comprehend the language within a story problem, represent the relationship between the two known whole-number quantities within the story, and finally execute a problem-solving strategy using addition, subtraction, multiplication, or division to identify the third unknown quantity.

One-Step Arithmetic Story Problem-Solving in General Education

Two factors that impact successful story problem-solving in general education are language comprehension and conceptual understanding of the story problem. Second-grade hearing children’s increased reading comprehension scores predicted increased story problem-solving correctness, both within the same school year and three to five school years later (Björn et al., 2016). Other studies have identified a positive correlation between reading comprehension and story problem-solving correctness in the second and fourth grades, meaning the higher a hearing child's reading comprehension score, the higher that child's story problem-solving correctness when story problems are presented in print (Fuchs et al., 2015; Fuchs et al., 2018; Vilenius-Tuohimaa et al., 2008). While correlation does not equal causation (i.e., it cannot be conclusively said increasing children's reading comprehension *will* increase their story problem-solving correctness), this correlation does show that reading comprehension is important in story problem-solving when presented in text. Similar studies presenting story problems in alternative forms, such as speaking or wordless pictures, were not identified.

Also important to story problem-solving is the ability to conceptually understand the relationships between quantities a story problem expresses. Skilled story problem-solvers represent the mathematical relationships within the story to find the answer, while unskilled story problem-solvers attempt to use numbers and keywords within the story to find the answer because they do not comprehend, and so cannot represent, the mathematical relationships inside the story (Garofalo, 1993; Hegarty et al., 1995).

In conclusion, the general education research base suggests reading comprehension and story problem-solving correctness are correlated and important to each other. However, these factors are not interchangeable; children must first have a minimal level of language comprehension to access the story problem before they can set about reaching conceptual understanding *through* the language they comprehend. Given this ordered process, the following discussion focuses on how research into d/hh children's story problem-solving has, and has not, addressed language comprehension.

Language Within One-Step Arithmetic Story Problem-Solving

The majority of studies regarding d/hh children’s story problem-solving have not adequately accounted for the impact of language comprehension upon story problem-solving (e.g., Antia et al., 2009; Edwards et al., 2013; Qi & Mitchell, 2011; Traxler, 2000), though one study did account for links between language comprehension and mathematics scores overall (Henner et al., under review). Studies have classified many d/hh children’s story problem-solving outcomes as below expected levels based on age or grade at preschool (Kritzer, 2009; Pagliaro & Kritzer, 2013), elementary (Hyde et al., 2003; Sarant et al., 2015), and high school levels (Hyde et al., 2003; Qi & Mitchell, 2011; Traxler, 2000). However, none of these studies measured (and thus accounted for) participant language comprehension leading to story problem access within their designs.

Given the critical importance of language comprehension to story problem-solving outcomes, as highlighted by the general education literature, it can be suggested that the field of deaf education is largely (though not entirely) unable to classify d/hh children’s one-step arithmetic story problem-solving appropriately because the question of language comprehension has not been adequately controlled for (as discussed below, studies examining d/hh children’s story problem-solving, while accounting for language comprehension do exist, but they are few and far between). Further complicating matters, many of these studies blended children of different primary languages and/or communication modes in their samples (i.e., ASL, listening/spoken/written English, and/or communication systems), making the impact of different languages and/or communication modes upon story problem-solving impossible to identify (e.g., Antia et al., 2009; Pagliaro & Kritzer, 2013; Qi & Mitchell, 2011; Traxler, 2000).

When studies about one-step arithmetic story problem-solving include such communicatively heterogeneous samples, findings may reflect only a portion of the sample's true skills. This is particularly true if the majority of the sample uses one language, possibly leading to findings that reflect the majority's story problem-solving outcomes while masking the minority's story problem-solving outcomes. Supporting the importance of sample homogeneity in mathematics research for d/hh children, several researchers have called for studies to disaggregate data across subpopulations, such as d/hh children who use sign language versus d/hh children who use listening/spoken language (Gottardis et al., 2011; Lange et al., 2013), or cochlear implant (CI) users versus hearing aid users (Thoutenhoofd, 2006). Some studies of d/hh learner's mathematics have already begun this disaggregation, either by including only one homogeneous sample (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012; Serrano Pau, 1995) or by reporting findings broken down by participant language use (Borgna et al., 2018).

Only two known studies, presented in three distinct reports of d/hh children's one-step story problem-solving, have controlled for language comprehension *and* sample homogeneity: Ansell and Pagliaro (2006) and Pagliaro and Ansell (2012), which analyzed the same dataset in two different ways, and Serrano Pau (1995). Both studies used measures of language comprehension and included communicatively homogeneous samples of children who only used one language and mode. Serrano Pau (1995) included d/hh children who used listening/spoken Spanish, using the Psychopedagogic

Instrumental Learning Test (Canals et al., 1988) as a standardized measure of reading comprehension. Ansell and Pagliaro (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012) included signing d/hh children, and conducted an ASL comprehension screening with three subtests drawn from the American Sign Language Assessment Instrument (ASLAI: Hoffmeister, 1999): (a) ASL Antonyms, (b) ASL Synonyms, and (c) Plurals and Arrangement. These screening subtests were chosen because a formal standardized measure of ASL had not yet been developed. The three subtests had “acceptable” internal consistency ratings (ranging from .70 to .87), indicating the subtest comprehension scores were a valid and reliable measure of ASL comprehension (Pagliaro & Ansell, 2012).

While these two studies are notable within the research base of d/hh children’s one-step arithmetic story problem-solving because they accounted for language comprehension *and* included communicatively homogeneous samples, no known studies of oral d/hh children’s story problem-solving were identified, suggesting the field of deaf education knows very little explicit information regarding their story problem-solving. The present study, describing the story problem-solving of oral d/hh children while accounting for language comprehension within a homogeneous sample, offers a clear initial picture of oral d/hh children’s story problem-solving, a picture that teachers, researchers, and stakeholders can unequivocally use to support oral d/hh children.

A body of literature does indicate a relationship between reading comprehension and critical access to text-based one-step arithmetic story problems. While reading comprehension is not the same as language comprehension, such findings indicate that comprehending story problem content is linked to story problem correctness. Hearing children with increased reading comprehension scores also have increased scores on story problem-solving tasks, whether they read the story problem independently (Riley et al., 1983; Vilenius-Tuohimaa et al., 2008), listen to an adult read it to them (Boonen et al., 2014), or read along while an adult reads aloud (Fuchs et al., 2015; Fuchs et al., 2018). Studies in deaf education have also linked d/hh children's and adult's increased reading comprehension scores to increased story problem-solving correctness when story problems are presented in written form at both the elementary (Hyde et al., 2003; Serrano Pau, 1995) and college levels (Kelly & Mousley, 1999, 2001). However, no studies were found that linked spoken language comprehension measures to one-step arithmetic story problem-solving given in that mode, in either general or deaf education.

Findings linking reading comprehension to story problem-solving cannot be applied to listening comprehension because literacy development studies with hearing children (Catts et al., 2003; Gottardo et al., 2018; Keenan et al., 2006; Lervåg et al., 2018) and oral d/hh children (Nelson & Crumpton, 2015; Spencer et al., 2003) have documented children having two different skill levels in reading comprehension and listening comprehension. The present study begins to address this gap in the deaf education literature by comparing oral d/hh children's spoken language (English) comprehension scores to analyses of their story problem-solving.

Oral D/HH Children: A Distinct Group of Learners

The following section describes why studies of hearing children's and signing d/hh children's one-step arithmetic story problem-solving cannot be assumed to represent oral d/hh children's one-step arithmetic story problem-solving. Understanding oral d/hh children as a distinct group of learners increases the importance of the present study. Oral d/hh children share common characteristics with their hearing and signing d/hh peers: they primarily use listening/spoken language like their hearing peers but have reduced access to sound like their signing d/hh peers.

Oral D/HH and Hearing

Despite oral d/hh children and hearing children both accessing the world through the same language modality, these populations differ greatly in how accurately they comprehend content presented through listening/spoken language, including one-step arithmetic story problems, Studies have reported only 30-50% of oral d/hh participants scoring within an age-appropriate range on standardized spoken language comprehension measures. One study included 39 oral d/hh children aged 5 to 14 years and measured their spoken English sentence comprehension using the *Test of Language Competence – Expanded*; results indicated only 30% of participants scored at or above age-level on this assessment, with the sample's mean score of 4.78 falling well below the expected range of 7 to 13 (Schorr et al., 2008). A second study included 70 oral d/hh children, aged 5 to 13 years, and measured their comprehension of an incomplete sentence to which the child supplied the final missing word by looking at a picture. Findings indicated only 50% of the oral d/hh children achieved age-appropriate or above-age-appropriate scores on this task (Boons et al., 2013). To clarify, Boons et al.'s (2013) study was conducted entirely in spoken Dutch, not English, but these findings indicate that even in languages other than English, oral d/hh children may not develop age-appropriate oral language comprehension from ages 5 to 13 years. While oral d/hh children are certainly capable of gaining age-appropriate language comprehension, reaching this potential is still not a reality for many. A recent study first assessed the English vocabulary and phonological comprehension skills of 42 oral d/hh children, aged 5 to 7 years, and then compared these scores to those of a comparable cohort collected 10 years earlier. Vocabulary comprehension scores improved but were still below chronological age level and phonological comprehension scores remained the same for all children across the decade (Harris et al., 2017). Given the relatively high percentage of oral d/hh children not meeting spoken language comprehension outcomes, any learning that stems from this skill including solving spoken one-step arithmetic story problems cannot be assumed to be similar in nature to their hearing peers'.

Oral D/HH and Signing D/HH

Although oral d/hh and signing d/hh children may be audiologically similar, each population uses a different language and language modalities to communicate "through the air." It has been well-established since the 1980s that signing d/hh children exposed to natural signed language early in life—particularly from birth—develop language along a pathway commensurate with their hearing peers, achieving age-appropriate levels of sign language prior to one year of age (Chen Pichler, 2012; Mayberry & Squires, 2006) which then carries on into their formal school years (Davidson et al., 2014; Tomaszewski et al., 2019). Thus, oral d/hh children cannot be assumed to have the same level of access or comprehension to spoken one-step story problems as their signing d/hh peers do to signed one-step story problems; for this reason, comparisons between the two populations should be made with caution.

Given that one-step arithmetic story problems are presented almost entirely through language (as opposed to numerical notation), language comprehension is foundational to one-step arithmetic story problem-solving. Yet, as stated above, oral d/hh children's language comprehension outcomes are distinct from both hearing children and signing d/hh children.

Thus, what is needed is research on oral d/hh children's story problem-solving to begin understanding how *they* approach this task given their unique auditory and linguistic contexts.

The Study's Conceptual Framework

This study was guided by two frameworks, designed by Carpenter et al. (2015), to provide teachers a structured way to think about their students' story problem-solving (Carpenter et al., 1989; Carpenter et al., 2015; Franke et al., 2001; Franke & Kazemi, 2001). One framework classifies types of story problems and the other classifies types of strategies used to solve story problems (Carpenter et al., 1999, 2015). Understanding the content of these conceptual frameworks and how they are related provides researchers and teachers powerful tools with which to consider children's one-step arithmetic story problem-solving.

History of the Frameworks

The present study's conceptual frameworks are drawn from Cognitively Guided Instruction (CGI; Carpenter et al., 2015). While studies of children's story problem-solving strategies have been conducted since the early 1980s (Carpenter & Moser, 1984), as series of seminal papers coalesced in the 1980s and early 1990s and refined this body of research into frameworks of story problem types and strategy types, using them to describe the story problem-solving strategies of hearing children from kindergarten to third grade. Below I describe four examples.

The first paper combined previous research that classified story problem types by use of addition or subtraction with other previous research classifying story problem types as containing action or no action and merged these frameworks to establish story problems into four types. Then, working with first-graders (N = 43), Carpenter et al. (1981) described the strategies children used to solve these four story problem types and concluded these young hearing children, in fact, possessed an extensive repertoire of strategies before formal story problem-solving instruction, a finding they stated that differed from past research which suggested that children possessed very few story problem-solving strategies (Carpenter et al., 1981).

A second paper gathered different examples of story problems from various research studies and arranged them into three primary types (Change, Combine, and Compare) and multiple subtypes (labeled by number; i.e., Change 3), distinguished by how the quantities within the story problems were related and whether such a relationship involved addition or subtraction (Riley et al., 1983). The third paper, a longitudinal study of hearing children in Grades 1-3, analyzed the strategies these hearing children used to solve different types of story problems within Riley et al.'s (1983) framework, and developed another framework describing the strategy types children used from Grades 1-3 to solve one-step arithmetic story problems (Carpenter & Moser, 1984). Finally, in

1993, Carpenter and his colleagues used this same framework to analyze hearing kindergarten children's story problem solving.

This research series contributed to the conceptual frameworks within this study in distinct ways. Carpenter et al. (1981) realized the present framework of strategy types was perhaps not extensive enough, and so proposed a strategy types framework similar to what is used in the present study from Carpenter et al. (2015)—the descriptive words (i.e., Join, Compare) are not identical in the 2015 work compared to the 1981 work, but the story problem structures are. Riley et al.'s (1983) work directly informed Carpenter and Moser's (1984) subsequent study that extended descriptions of story problem-solving strategy use from Grades 1-3. The most recent compilation of story problem types and strategy types, as informed by this body of literature, is found in Carpenter et al. (2015) and forms the basis for the present study.

The Flexibility of the CGI Frameworks

The CGI frameworks have been implemented in various general education classrooms that include the natural myriad of children that comes within that context. These include classrooms of hearing children that may or may not have included students receiving services for specific learning disabilities (no details are given) (e.g., Carpenter et al., 1993; Carpenter et al., 2015; Carpenter & Moser, 1984; Moscardini, 2014), teacher preparation (Bright & Vacc, 1994; Jacobs et al., 2017; Swars et al., 2009), classrooms with mathematically gifted third- through fifth-graders (McComas, 2011), rural elementary schools (Guerrero, 2014), and urban school classrooms with disadvantaged students from minoritized groups; that is, African-American, Hispanic, or Native

American students living in low SES households (Villaseñor & Kepner, 1993). Implementing the CGI frameworks in a wide range of settings as seen here speaks to their adaptability and ultimately usefulness. However, this research base only provides general indications of one-step arithmetic story problem-solving development for the general population. Until 2006, indications for how d/hh children solved one-step arithmetic story problems through these conceptual frameworks and concepts were unknown.

Use of CGI in Special Education (Outside of Deaf Education)

Studies using the CGI framework with students who had specific disabilities (i.e., blind/low vision, autism, dyscalculia, mobility disorders, etc.) were not identified, and none of the above studies accounted for students receiving special education supports within their analyses. One study that did account for special educational needs did not specify which special educational needs were present in the study. This analysis examined how providing two CGI workshops to Scottish elementary teachers impacted their inclusion and support of children with and without special educational needs in their mathematics lessons (Moscardini, 2014). These special educational needs were not defined beyond that a segment of the class required either additional one-on-one support in class or pull-out support with an educational assistant. Findings were reported in terms of teacher instructional practices and teacher observations of student abilities. After the teachers attended multiple CGI workshops and implemented CGI in their classrooms there was a documented decrease in separate one-on-one instruction and an increase in the inclusion of children needing special mathematics support; multiple teachers expressed surprise at what some of the mathematically struggling students knew and could do when given the chance to simply share ideas in a group. Teachers also reported a deeper understanding of students’ skills and knowledge. For example, one teacher reported that before the CGI workshops they would have simply concluded that two students could proficiently add. Yet, by observing these students’ thinking through the CGI frameworks, one student in fact possessed stronger counting skills and addition skills than the other. Finally, multiple teachers recognized that by using the CGI frameworks to observe struggling mathematics learners they sometimes witnessed these students demonstrating a level of mathematical understanding well beyond their capabilities as formally documented on standardized tests and special education planning materials (Moscardini, 2014).

These findings support the applicability of the CGI framework within special education environments. This framework can adapt to different students’ paths and rates of development, being successfully used with students who are not necessarily progressing mathematically at the same rate as typical children. Also, using this framework led to positive programming changes for struggling mathematics learners, including more frequent recognition of their true mathematics skill, sometimes beyond formally-documented expectations, and more time spent in the classroom with their typical peers. To date, no known studies have brought the CGI framework into the classrooms of d/hh children, using it to support their teachers’ mathematics instruction as described here in Moscardini (2014). However, the CGI frameworks *have* been used to describe the story problem-solving of d/hh children who have age-appropriate ASL comprehension (signing d/hh).

Use of the CGI Frameworks with Signing D/HH Children

In this work, researchers used the CGI frameworks to describe similarities and differences in one-step arithmetic story problem-solving between signing d/hh children and Carpenter et al.'s (2015) descriptions of hearing children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). The story problem-solving of signing d/hh children and hearing children were similar in two ways: (a) patterns of viable strategy use, and (b) patterns of flexible strategy use. Frequency counts and Chi-square analyses of signing d/hh children's viable strategy use indicated children who had higher ASL comprehension scores were significantly ($p < .000$) more likely to use more viable strategies (Pagliaro & Ansell, 2012). Unless a student guesses, and is lucky, selecting a viable strategy requires comprehending the story problem such that the relationships between quantities in a story problem are understood, because only viable strategies represent this relationship correctly. That signing d/hh children with higher ASL language comprehension selected viable strategies more often suggests their higher language comprehension led to higher story problem access and comprehension, supporting the logical statement that language comprehension is critical to successful one-step arithmetic story problem-solving regardless of one's primary language because language is what provides initial access to the relationship between quantities described in the story problem.

Signing d/hh children, like their hearing peers, also demonstrated a pattern of flexible strategy use (Pagliaro & Ansell, 2012). Signing d/hh children were divided into "more" and "less" successful problem-solvers based on how many story problems they solved using a viable strategy, with more successful problem-solvers using more viable strategies. Findings showed more successful problem-solvers used modeling strategies more often than counting strategies on more difficult story problems. This shift in strategy use, in response to relative story problem difficulty, indicates more successful problem-solvers were more flexible in their strategy use (Pagliaro & Ansell, 2012). Thus, it is currently understood that young children, regardless of language or hearing acuity, adjust the strategy types they use in response to how easy or difficult a story problem is.

In contrast, the story problem-solving of signing d/hh children and hearing children differed in three ways: (a) the types of strategies they used more often, (b) the types of viable strategies they used, and (c) their relative story problem difficulty (Ansell & Pagliaro, 2006; Carpenter et al., 2015; Pagliaro & Ansell, 2012). In terms of general strategy use (viable and non-viable), Pagliaro and Ansell (2012) found that signing d/hh children employed counting strategies more than modeling or fact-based strategies. In their discussion the researchers suggested counting strategies were used most often perhaps because counting fits within the grammatical structure of ASL in a way that it does not fit spoken languages (Pagliaro & Ansell, 2012). In contrast, hearing children only begin using counting strategies more often than modeling strategies after they recognize they need not represent every quantity in a story problem to solve it (Carpenter et al., 2015). This suggests that language use—verbal/auditory (English) or visual/spatial (ASL)—may impact the types of strategies children are more likely to use.

However, while the signing d/hh children used counting strategies more often overall, they were more likely to use a viable modeling strategy than a viable counting strategy on more difficult story problem types (Pagliaro & Ansell, 2012). Hearing children, it is generally understood, do not display such a distinction between the strategy types they use most often and the strategy types they are most likely to use viably (Carpenter et al., 2015).

Differences were also found between signing d/hh and hearing children's relative story problem difficulty. Of the six story problems types analyzed, Ansell and Pagliaro (2006) found easier story problems involved summing quantities together while more difficult story problems removed one set from another (Ansell & Pagliaro, 2006). Carpenter et al. (2015), on the other hand, suggested hearing children find story problems easier when the language of the story problem includes explicit action giving children guidance in "doing what the problem says to do." This difference indicating what makes a story problem relatively easy or difficult is related to a child's language use, hearing level, or possibly both.

This body of research ultimately indicates that signing d/hh children and hearing children approach story problem-solving in similar and different ways. Signing d/hh children do not approach story problem-solving as simply "hearing children who cannot hear" (Marschark, 2014, "Implications," para. 1), but neither are they so different from hearing children that their story problem-solving must be considered an entirely distinct process. How oral d/hh children fit into this context is currently unknown. The present study, for the first time, uses the CGI frameworks to examine how oral d/hh children solve story problems. Using this specific framework allows for comparisons between research from deaf and general education, further clarifying how all children solve story problems and providing a context for understanding how oral d/hh children solve story problems relative to their auditory and linguistic peers.

Conceptual Frameworks of CGI

Story Problem Types

The first of Carpenter et al.'s (2015) conceptual frameworks is the framework of story problem types. Table 1 illustrates the relationships between the different story problem types. To provide context, an example of each type of story problem is provided. To aid in comparing problem types the content of the example stories will be the same: all stories will feature Lisa, Amy, and either 3, 5, 8, or 15 flowers. This framework includes 14 semantically different types of one-step arithmetic story problems. Each story problem contains three quantities—two known and one unknown. To solve the story problems, children (and adults) use both known quantities to find the third unknown quantity. These story problems are distinguished from one another based on two factors—how relationships between quantities are described, and the location of the unknown quantity.

Table 1

Story Problem Types in CGI

<table>
<tr><th>Problem Type</th><th colspan="3">Position of Unknown Quantity</th></tr>
<tr><td>Join</td><td>Result
Lisa has 3 flowers. Amy gives her 5 more flowers. How many flowers does Lisa have now?</td><td>Change
Lisa has 3 flowers. Amy gives her some more flowers. Now Lisa has 8 flowers. How many flowers did Amy give Lisa?</td><td>Start
Lisa has some flowers. Amy gives her 5 more flowers. Now Lisa has 8 flowers. How many flowers did Lisa have at the start?</td></tr>
<tr><td>Separate</td><td>Result
Lisa has 8 flowers. She gives 5 to Amy. How many flowers does Lisa have now?</td><td>Change
Lisa has 8 flowers. She gives some to Amy. Now Lisa has 3 flowers. How many flowers did Lisa give Amy?</td><td>Start
Lisa had some flowers. She gave 5 to Amy. Now Lisa has 3 flowers. How many flowers did Lisa have at the start?</td></tr>
<tr><td>Part/Whole</td><td colspan="2">Whole
Lisa has 3 purple flowers and 5 blue flowers. How many flowers does Lisa have?</td><td>Part
Lisa has 8 flowers. 5 are purple and the rest are blue. How may blue flowers does Lisa have?</td></tr>
<tr><td>Compare</td><td>Difference
Lisa has 5 flowers. Amy has 3 flowers. How many more flowers does Lisa have than Amy?</td><td>Compare Quantity
Lisa has 5 flowers. Amy has 3 more flowers than Lisa. How many flowers does Amy have?</td><td>Referent
Lisa has 5 flowers. She has 3 more flowers than Amy. How many flowers does Amy have?</td></tr>
<tr><td rowspan="2">Grouping</td><td colspan="3">Multiplication
Lisa has 5 bouquets. Each bouquet has 3 flowers. How many flowers does Lisa have?</td></tr>
<tr><td colspan="2">Partitive Division
Lisa has 15 flowers. She wants to make 5 bouquets, with the same number of flowers in each bouquet. How many flowers will be in each bouquet?</td><td>Measurement Division
Lisa has 15 flowers. If she puts 3 flowers in each bouquet, how many bouquets will she make?</td></tr>
</table>

Note. Source: Carpenter et al., 1993; Carpenter et al., 1999, 2015.

Relationships Between Quantities

Specifically, the language of Join/Separate story problems indicates the action of combining or removing quantities. The next two story problem types (Part-Part-Whole and Compare) describe a relationship between quantities, relating either one set to its subset (Part-Part-Whole) or comparing two distinct sets (Compare); neither explicitly describe combining or removing quantities. In the final story problem type, Grouping, quantities are described as being in multiple groups, either a specific number of groups or a specific number within each group.

Location of the Unknown

In Join and Separate story problems the unknown can be in one of three positions in reference to the action described within, either at the start, the change, or the result (Carpenter et al., 1999, 2015). In Part-Part-Whole story problems, either one of the two parts is unknown (part unknown) or the whole is unknown (whole unknown). All Compare story problems identify two sets of items and their relationship to each other. In Compare Difference Unknown story problems, the size of both sets is known and the amount by which one set exceeds the other is unknown; in Compare Quantity Unknown and Compare Referent Unknown story problems, just one of the sets is known as well as the difference between the two sets, and the other set is unknown. The distinction between Quantity Unknown and Referent Unknown lies in which sets are referenced: in in Quantity Unknown one set (unknown) is compared to a second set (known) that increases or decreases by a known amount (the difference value), while in Referent Unknown one set (known) is compared to itself along with the increase or decrease of the difference value to determine the size of the second set (unknown). (Carpenter et al., 1999, 2015).

Finally, in Grouping story problems, either the total quantity of objects is unknown (multiplication), the number of groups is unknown (measurement division), or the number of objects in a group is unknown (partitive division; Carpenter et al., 1999, 2015). These 14 story problems comprise one of the conceptual frameworks Carpenter and his colleagues (Carpenter et al., 1999, 2015) have built to analyze hearing children's one-step arithmetic story problem-solving.

Strategy Types

The second conceptual framework of CGI (Carpenter et al., 2015) relates to the strategies hearing children use to solve one-step arithmetic story problems. These strategies define the ways children represent the mathematical relationship inside the story, varying in how concretely or abstractly they achieve this representation. While there are many specific strategies that children use (e.g., counting on larger, counting on first, etc. See Carpenter et al., 2015 for full explanation), these can be categorized under three main strategy types: modeling, counting, and fact-based. These main strategy types will be referenced in the present study. The strategy types move from concrete to abstract—modeling, counting, and fact-based respectively, and can be broadly ordered to describe hearing children's story problem-solving. In a modeling strategy, both known quantities are physically represented (e.g., using cubes, fingers, drawing, etc.) and acted upon. More specifically, in a *direct* modeling strategy, both quantities are physically represented and they are manipulated to follow the action within the story problem. In the context of this study, “modeling strategies” included both modeling and direct modeling strategies. Counting strategies use one known quantity to find the starting point on the counting string (i.e., 0,1, 2, 3, 4, 5 . . .) and uses the other known quantity count forwards or backwards, up or down the counting string. Fact-based strategies represent the quantities and the relationships between them mentally, without using drawings, cubes, fingers, etc. to represent any known quantities.

Like the story problem types conceptual framework, this strategy framework is also not meant to represent hearing children’s story problem-solving development along a rigid trajectory. The progression from modeling to counting to fact-based strategies is not linear and is dependent upon the type of story problem and the size of quantities in the story problem. Over time, hearing children tend to use counting strategies more than modeling strategies, eventually typically using fact-based strategies more often than counting or modeling strategies (Carpenter et al., 2015). This happens because children rely less and less on external representations to represent the quantities and relationships between quantities in a story problem (Carpenter et al., 2015).

Hearing children use these strategy types flexibly, adjusting when story problems are made more difficult either because of the story problem type and/or the size of quantities (e.g., 5 versus 15). For example, a hearing child might predominantly use counting strategies, but if asked to solve a story problem they find difficult may instead use a modeling strategy (Carpenter & Moser, 1984). This is because modeling strategies are considered the most concrete, representing all known quantities in the story problem; counting strategies are considered slightly more abstract, because while the physical quantities are not represented the movement along the counting string is. Fact-based strategies are considered the most abstract because all of the representations of known quantities, and the relationship between them, are mental. Understanding how the story problem types and strategy types fit together leads to two further concepts important for understanding and describing children's story problem-solving—strategy viability and relative story problem difficulty.

Strategy Viability

Strategy viability indicates whether a specific strategy could lead to the correct answer (Carpenter & Moser, 1984; Carpenter et al., 2015; Pagliaro & Ansell, 2012). A viable strategy will lead a child to the correct answer so long as the child performs the final arithmetic calculation correctly (Carpenter et al., 2015; Carpenter & Moser, 1984; Pagliaro & Ansell, 2012). Using a non-viable strategy, a child could not possibly reach the correct answer even if no miscounting or miscalculation occurs (Carpenter et al., 2015; Carpenter & Moser, 1984; Pagliaro & Ansell, 2012). For example, consider this story problem: *Lily has three flowers. Mom gives her two more flowers. How many flowers does Lily have?* If the child combines three cubes and two cubes, but miscounts and answers six, the strategy is still viable. On the other hand, if the child begins with three cubes and removes two, even if they answer 1 the strategy is non-viable.

Relative Story Problem Difficulty

Relative story problem difficulty indicates how easy or difficult a specific story problem is for a child or group of children. As reported by Riley et al. (1983), some past studies in general education drew conclusions regarding relative story problem difficulty using the percentage of correct answers children gave. That is, the more often a type of story problem was solved accurately, the easier it was said to be; conversely, the less often a type of story problem was solved accurately, the more difficult it was said to be.

More recent general education studies defined relative story problem difficulty instead by the number of viable strategies children used (Carpenter et al., 1993; Carpenter et al., 1999, 2015; Carpenter & Moser, 1984). Similarly, in deaf education, some have analyzed d/hh children's relative story problem difficulty using measures of story problem-solving correctness (Frostad, 1999; Hyde et al., 2003), while others have used percentage of viable strategy use (Ansell & Pagliaro, 2006).

Defining relative story problem difficulty by percentages of viable strategy use is more appropriate than using percentages of correctness because findings can inadvertently be attributed to miscounting or miscalculating when correctness is used. For example, consider the following story problem: *Lisa has three flowers. Amy has five flowers. How many flowers do Lisa and Amy have?* One child creates sets of three flowers and five flowers, combining them to give eight as the answer. Another child creates sets of three and *four* flowers, combining them to give seven as the answer. Defining relative story problem difficulty using correctness would suggest one child found this story problem easier than the other child, even though both children comprehended the relationships between quantities in the story problem. Defining relative story problem difficulty by viable strategy use would recognize that both children comprehended the relationships between quantities in the story problem, concluding both children found this story problem equally easy. Thus, viable strategy use as a measure of relative story problem difficulty connects difficulty to story problem comprehension, and not inadvertently to an unidentified factor such as a miscount or miscalculation.

This concept of strategy viability indicates a story problem's relative difficulty because a viable strategy indicates the child conceptually understood the relationships between quantities within the story problem. Suppose 80 out of 100 children used a viable strategy to solve Story Problem A, but only 30 out of those 100 children used a viable strategy to solve Story Problem B. Story Problem A is relatively easier (for the sample) than Story Problem B because more children conceptually understood the relationships within Story Problem A, as suggested by selecting a viable strategy. It is important to note that selecting a viable strategy *indicates*, does not confirm, that a child comprehended the relationships between quantities in a story problem; sometimes children make lucky guesses, selecting a strategy because they feel confident carrying it out, or they recently learned about it. Researchers have used frequencies of viable strategy use to identify relative story problem difficulty for signing d/hh children (Ansell & Pagliaro, 2006) and for hearing children (Carpenter et al., 2015).

In conclusion, the CGI frameworks are designed to give teachers and researchers a place to begin understanding what children might do and then personally add on from there (Carpenter et al., 2015; Franke et al., 2001). There are documented cases where teachers added levels and details to the CGI conceptual frameworks through their own teaching practice, making the framework they used more complex and customized (Franke et al., 2001). The CGI conceptual frameworks do not prescribe "one-size-fits-all" but rather the notion of "a place to start" and add to a given particular context. Adopting such a flexible framework allowed patterns unique to oral d/hh children's one-step arithmetic story problem-solving to emerge. Furthermore, using Carpenter et al.'s (2015) frameworks supports the comparative nature of the present study.

Purpose of the Study

The study's initial impetus was the absence of known research on how oral d/hh children solve one-step arithmetic story problems. An investigation into signing d/hh children's story problem-solving used the CGI frameworks, allowing a comparison between signing d/hh children and hearing children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). Utilizing these same frameworks extends the comparison between signing d/hh and hearing children to a new subpopulation within deaf education—oral d/hh children. This extension is critical for two reasons.

First, studies have documented that when teachers use the CGI frameworks to observe and analyze hearing children's one-step arithmetic story problem-solving, their story problem-solving correctness and viable strategy use increased (Carpenter et al., 2000; Carpenter et al., 1989; Fennema et al., 1996; Viallseñor & Kepner, 1993). Another study reported that using the CGI framework improved mainstream classroom teachers' understanding of how their students with special educational needs solved story problems, leading to those children being more included in mathematics lessons and also being given opportunities to demonstrate mathematics knowledge beyond what adults thought they were capable of doing (Moscardini, 2014). Collectively, these findings suggest CGI has the potential to support teachers trying to develop d/hh children's story problem-solving. The present study ensures researchers and teachers wanting to use the

CGI framework can immediately support either oral d/hh children or signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012) using a description of one-step arithmetic story problem-solving that is directly applicable to the population with which they are working.

More importantly, this study further answers a call from researchers in deaf education for studies in mathematics that account for sample homogeneity (Gottardis et al., 2011; Lange et al., 2013). While past research with signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012) and Spanish speaking d/hh children (Serrano Pau, 1995) began addressing this call, the present study fills this gap further with English-speaking d/hh children.

Finally, oral d/hh children are a unique group, straddling both the deaf and hearing worlds. They are distinct from signing d/hh children due to their listening/spoken language use, yet similar because they too have hearing loss. Oral d/hh children are also distinct from hearing children due to their hearing loss, yet they too access the world (including story problems) through listening/spoken English. How these two interacting factors—hearing acuity and listening/spoken English use—impact children's story problem-solving is currently unknown. The present study will begin a conversation with teachers, researchers, and stakeholders to understand oral d/hh children's story problem-solving.

Theoretical Framework

Constructivism

The study's theoretical framework is based on constructivism. Constructivism emerged out of the work of Piaget, Bruner, and Goodman (Ertmer & Newby, 2013) and is built upon four assumptions about learners and learning. First, the world is not an external entity waiting to be discovered but is instead defined by each individual person based on personal experience (Bruner, 1985). Second, learning develops when an experience cannot be accommodated into one's current understanding; this disequilibrium requires a new definition of reality in which learners reorganize their understanding to subsume the old definition and accommodate the new one, a process which is continually open to change (Bruner, 1985; Cobb, 1994). Third, learners apply new information about a topic to what they already know about that topic, building new knowledge on top of old knowledge (Kanselaar, 2002). Finally, while constructivism describes how knowledge develops, it does not prescribe one specific program or process necessary for learning to occur (Simon, 1995). The researcher, as a learner, will operate within these four assumptions using the CGI (Carpenter et al., 2015) conceptual frameworks (story problem types, strategy types) and concepts (strategy use, strategy viability, relative story problem difficulty) as guides to develop an initial description of oral d/hh children's one-step arithmetic story problem-solving.

Constructivism was chosen as the theoretical framework for this study because the study's conceptual frameworks (story problem types and strategy types), and the research documenting their usefulness to teachers and stakeholders wanting to understand more of children's one-step arithmetic story problem-solving (e.g., Carpenter et al., 2000; Carpenter et al., 1989; Carpenter & Moser, 1984; Fennema et al., 1996) were also grounded in constructivism. To use these theoretical frameworks as they were intended, a constructivist approach was necessary.

Another related benefit to using constructivism in the present study was the allowances made for the simultaneous growth of new knowledge on old knowledge (Kanselaar, 2002) and the freeing lack of prescribed procedures for learning (Simon, 1995). From these allowances, data collection and statistical analyses were informed by frameworks from past research, but resulting findings were not compared to past research until after analyses were complete. These approaches led to an understanding of oral d/hh children's story problem-solving that was grounded in oral d/hh children's work, and not inadvertently understood through findings regarding another population's (i.e., hearing, signing d/hh) story problem-solving.

Constructivism has been criticized by Kirschner et al. (2006) and Phillips (1995). Kirschner et al. (2006) agree with constructivists that children build their own knowledge, but they disagree that the most effective form of instruction is having learners build their own understanding. Rather, Kirschner et al. (2006) state the most effective form of instruction is one knowledgeable person providing a less knowledgeable person with a model or example from the outset, a perspective taken from behaviorism (Schunk, 2016).

For example, adults bring mathematical experiences and background knowledge to one-step arithmetic story problem-solving that children may not have, allowing adults to mathematically transform the task in ways young children may not understand. In the opening chapter of Carpenter et al.'s (1999) textbook, the researchers provided three one-step arithmetic story problem types: Separate (result unknown), Join (change unknown), and Compare (difference unknown). Then the researchers stated, "most adults would solve all three of these problems by subtracting three from eight. To young children, however, these are three different problems" (p. 1). In this situation, if an adult showed a child how they would approach a story problem, without seeking to understand the child's approach, an opportunity to build upon the child's current knowledge would be missed. Carpenter et al. (1993) and Carpenter et al. (1999, 2015) illustrate this point in their works.

This distinction between adult conceptualizations and child conceptualizations of the relationships between quantities within a story problem may not have been recognized if Carpenter and his colleagues had used more experienced adult conceptualizations of story problem-solving to learn about children's conceptualizations. Had this occurred, the field would not know what children think about story problem-solving; rather, the field would know what adult thoughts about story problem-solving looked like when imposed upon children. Kirchner et al.'s (2006) disagreement with constructivism, that learning is most effective when others provide models for a learner to create knowledge through, does not provide theoretical space to understand someone's learning by what *that learner* understands or knows. Taking Kirchner et al.'s (2006) criticism into account within the study could have introduced ambiguity and bias into the results because findings would not be drawn directly from the story problem-solving of oral d/hh children. The researcher, to avoid this, consciously applied past findings from parallel research with hearing (Carpenter et al., 2015) or signing d/hh (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012) children only *after* findings regarding oral d/hh children were developed. This was critical so that the story problem-solving of oral d/hh children was not inadvertently understood entirely through another population's (i.e., hearing, d/hh children who use ASL) conceptualizations of story problem-solving.

The second criticism of constructivism, raised by Phillips (1995), states that constructivists are in danger of drawing conclusions through relativism, the idea that because knowledge is created within people it is only relative to the context in which it was created and does not need to be justified by any external absolute truths. Phillips (1995) goes on to argue that findings not evaluated against any external truths can contain unrecognized flaws. Such unrecognized flaws have implications for how closely findings represent the truth, as related to the learner. A paper discussing the dangers of relativism within special education research states, "the truth of a statement is not determined by who makes it but how it corresponds to objective evidence" (Kauffman & Sasso, 2006, pp. 85). That is, a finding is not true purely because one person believes or understands it; a finding is true in how it relates to knowledge external to the learner.

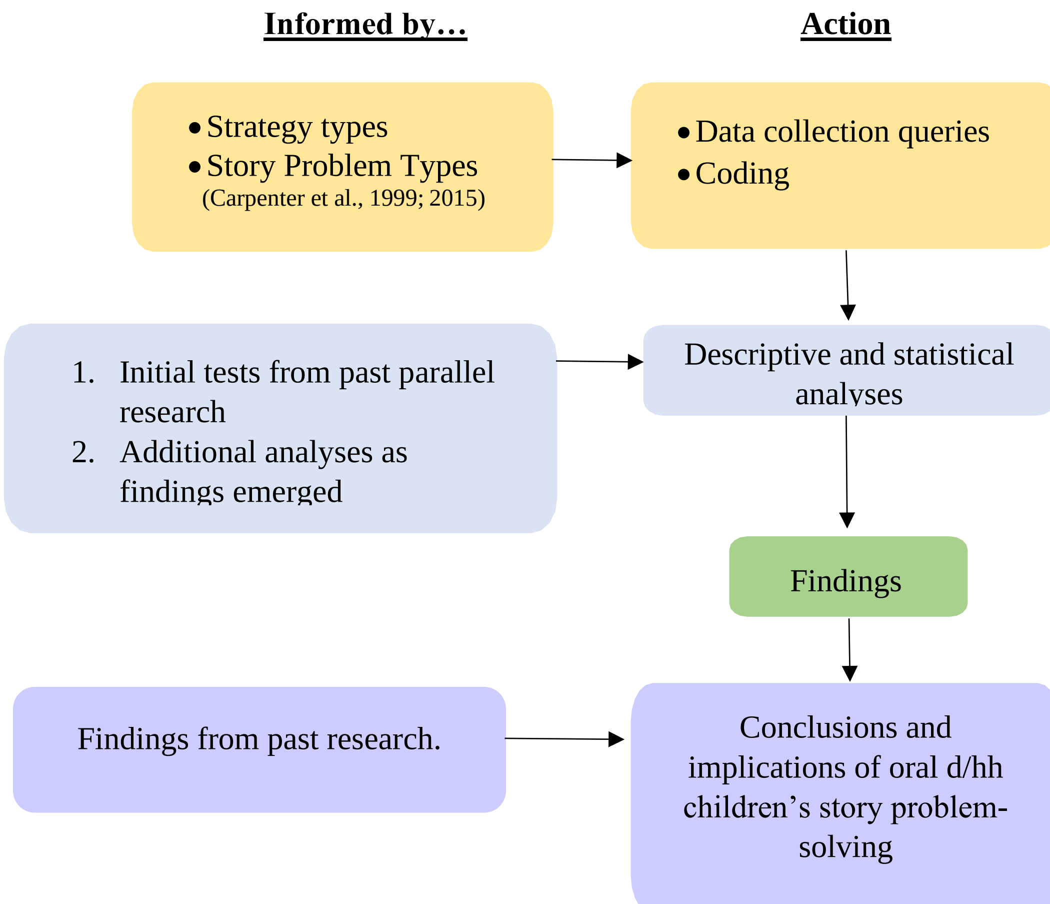
Working from this idea, for the study to accurately (truthfully) describe the story problem-solving of oral d/hh children through constructivism, ensuring findings were evaluated against objective evidence external to the researcher was critical. Conscious use of conceptual frameworks and concepts, research literature, and rigorous methodology reduced the danger of relativistic findings while still ensuring that data answering the research questions described how oral d/hh children solved the story problems. First describing oral d/hh children's story problem-solving using the conceptual frameworks, as well as descriptive and statistical analyses, before comparing findings to parallel research with other populations, accounted for the possibility of relativism. This ensured the findings regarding oral d/hh children's story problem-solving were relevant and practical for the fields of general and deaf education.

Combining Conceptual and Theoretical Frameworks

The final framework for this study combines the theoretical (Constructivism) and conceptual frameworks (CGI) as shown in Figure 1. The researcher used the conceptual frameworks to guide questions during data collection and coding. Statistical tests were initially selected because they had been used in past parallel studies (i.e., Ansell & Pagliaro, 2006; Carpenter et al., 2015; Pagliaro & Ansell, 2012), as part of building new knowledge on old knowledge. Further analyses were added, beyond the scope of past research, and were accepted because constructivism states that learning should not be contained within a single prescriptive process. In this way, findings were not evaluatively compared to other sources in any way until they were finalized, ensuring they represented oral d/hh children's one-step arithmetic story problem-solving as closely as possible.

Figure 1

The Theoretical Framework



General education research suggests that when teachers understand how children approach one-step arithmetic story problem-solving, their responses are increasingly targeted and, as a result, children's one-step arithmetic story problem-solving correctness and viable strategy use increases (Carpenter et al., 2000; Carpenter et al., 1989; Fennema et al., 1996; Viallseñor & Kepner, 1993). Understanding oral d/hh children's arithmetic story problem-solving may help teachers and researchers approach their concerning outcomes through effective instruction. The present study is guided by the following research questions.

Research Questions

The following research questions guided the study's design and data analysis. These questions sought to account for the factors of language comprehension, sample homogeneity, strategy use, and relative story problem difficulty.

1. What patterns of viable and non-viable strategies are used by d/hh children, kindergarten through Grade 3, who use listening/spoken English as their primary communication?
2. What is the relative difficulty of one-step arithmetic story problems for d/hh children, kindergarten through Grade 3, who use listening/spoken English as their primary communication?
3. How do patterns of strategy use and relative story problem difficulty compare between d/hh children, kindergarten through Grade 3, who use listening/spoken English as their primary communication and have age-appropriate English comprehension or higher, and those who have below-age-appropriate English listening comprehension?

CHAPTER III

METHOD

Introduction

Much of the research in deaf education suggests that children who are deaf or hard of hearing (d/hh) do not achieve mathematics outcomes commensurate with their hearing peers on standardized tests (e.g., Kritzer, 2009; Pagliaro & Kritzer, 2013; Qi & Mitchell, 2011; Traxler, 2000). One specific mathematical skill highlighted in this literature as being particularly difficult is arithmetic story problem-solving (e.g., Edwards et al., 2013; Hyde et al., 2003; Kritzer, 2009; Pagliaro & Kritzer, 2013). Specific information about story problem-solving within deaf education is limited, however, for several reasons. First, there is little research on mathematics with d/hh children in general. Second, much of the existing research includes communicatively heterogeneous samples (e.g., sign language, spoken language, and communication systems), limiting in-depth understanding of and pedagogical suggestions for each subgroup's outcomes within the sample. Few studies have focused on specific subgroups of d/hh students solving story problems. Studies that did focus on specific subgroups included two with d/hh children who used a signed language and one that included d/hh children who used listening/spoken Spanish. These studies suggested there may be a relationship between language use or proficiency and story problem-solving performance. However, no known studies have described the story problem-solving strategies of d/hh children who use listening/spoken English as their primary communication method. The following research questions guided this study in addressing this gap:

1. What patterns of viable and non-viable strategies are used by deaf/hard-of-hearing children, kindergarten through Grade 3, who use listening/spoken English as their primary communication?
2. What is the relative difficulty of one-step arithmetic story problems for deaf/hard-of-hearing children, kindergarten through Grade 3, who use listening/spoken English as their primary communication?
3. How do patterns of strategy use and relative story problem difficulty compare between deaf/hard of hearing children, kindergarten through Grade 3, who use listening/spoken English as their primary communication and have age-appropriate English comprehension or higher, and those who have below-age-appropriate English listening comprehension?

Study Design

The study is descriptive and quantitative, describing the story problem-solving correctness, relative story problem difficulty, and patterns of strategy use for oral d/hh. This section describes the design of the study, including recruitment procedures, inclusion/exclusion criteria, calculation of target sample size, data collection instruments and screening measures, data collection environment, procedures, coding, inter-rater agreement, and data analysis.

Target Sample Size

The target sample included a maximum of 54 d/hh children, kindergarten to Grade 3, who use listening/spoken English for their primary communication, with a minimum of five children from each grade level. This size was established by considering the statistical requirements of the three planned statistical analyses: descriptive statistics, chi-square, and ANOVA. The scant literature regarding sample sizes suggests for descriptive statistics that either any sample size is acceptable (Israel, 1992), or the sample must include at least 10 participants to ensure meaningful percentages that generate distinct patterns within the data (McCrum-Gardner, 2008). Multiple methodologists agree that the minimum number of data points for a chi-square analysis is five points per comparison (i.e., cell; McCrum-Gardner, 2008; Van Voorhis & Morgan, 2007; Welkowitz et al., 2011). For ANOVA, a target sample size of 54 was calculated by entering four factors into the computer program G*Power (Version 3.0.10)—the alpha level, the number of groups being compared, the effect size, and the power of the test. The relevant scholarly literature in deaf education and special education further supported the target sample size of 54. Research on the story problem-solving of signing d/hh had a final sample of 59 children with age-appropriate ASL comprehension (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). Research on the story problem-solving strategies of children with arithmetic learning disabilities included a sample of 60 children (González & Espinel, 2002).

Final Sample Size

This study included a final sample size of 24. Thirty-four oral d/hh children initially enrolled in the study. The datasets of 10 children were removed for the following reasons: (a) Seven were unable to access at least half the story problems through the computer videos as described in the data collection protocol (two participants required five or more story problems to be read aloud by the researcher and five required five or more story problems to be provided using sign supported speech); (b) One child did not attempt the first four story problems; and (c) Two did not meet participant inclusion criteria. Data are thus drawn from the solutions of 24 children as they attempted to solve up to nine story problems.

The planned data analyses for this study included using ANOVA to examine patterns within and across grade levels. However, given the small sample size, the data are no longer broken down by grade level, and ANOVA tests were not run. While this change results in more general findings, it also ensures findings are drawn from appropriate statistical measures. The smaller-than-expected sample size neither affected the validity of descriptive statistics nor Chi-square analysis, so these planned analyses were carried out as presented in Chapter III.

Recruitment

Participants were recruited in a two-phase process. The first phase used chain-referral sampling to identify teachers and parents of young d/hh children, who then recommended specific children as potential participants using inclusion/exclusion criteria (Penrod et al., 2003). Multiple methodologists have classified chain-referral sampling as a variant of snowball sampling (Biernacki & Waldorf, 1981; Goodman, 2011; Noy, 2008; Penrod et al., 2003). Like snowball sampling, chain-referral sampling identifies participants through the researcher's contacts (Biernacki & Waldorf, 1981; Noy, 2008; Spreen, 1992). Unlike snowball sampling, chain-referral sampling identifies participants through gatekeepers instead of directly contacting participants themselves (Goodman, 2011; Penrod et al., 2003). Chain-referral sampling intentionally samples from initial contacts who vary based on geographical location and demographic factors in an attempt to increase sample representativeness (Etikan et al., 2016). Using this method for the study allowed the researcher to build a sample of adequate size to represent demographic variations within the target population. Constructing such a sample was challenging given that hearing loss is a low incidence disability (Mitchell & Karchmer, 2004), and this study examined only a portion of that population based on both grade level (kindergarten – Grade 3) and language use (listening/spoken English).

The researcher, to establish this initial contact, distributed a "call to participate" notice, asking that it be forwarded on to teachers and parents. The "call to participate notice" clearly defined the expected disruptions, time frames, and resources required for participation, information that is important for encouraging gatekeeper buy-in (Devers & Frankel, 2000; Joseph et al., 2016). The document also stated and defined the inclusion/exclusion criteria to minimize the number of "false starts" (Biernacki & Waldorf, 1981, p. 149) recommended participants thought to be appropriate for the study who ultimately do not meet inclusion/exclusion criteria (see below). Defining the inclusion/exclusion criteria helped reduce sampling bias as the researcher and gatekeepers both understood which families, organizations, and programs would be appropriate or inappropriate to approach (Berniecki & Waldorf, 1981; Browne, 2005). This "call to participate" notice was distributed to multiple programs and agencies (i.e., universities, state education programs, schools, and professional organizations across multiple states) in an attempt to reduce sampling bias by increasing the probability that recommended participants were educated under multiple teachers/therapists and raised within multiple backgrounds, potentially increasing the representativeness of the sample.

After sending out the "call to participate" notice, a second recruitment phase—purposive sampling—guided the confirmation of suggested participants. Purposive sampling is particularly appropriate when the researcher is seeking to construct a homogeneous sample (based on single or multiple factors) by ensuring participants possess specific characteristics (Etikan et al., 2016; Ritchie et al., 2014). Purposive sampling is often compared to convenience sampling because neither methodology samples randomly (e.g., Battaglia, 2008; Etikan et al., 2016). However, convenience sampling selects participants solely based on availability (Battaglia, 2008; Etikan et al., 2016). In contrast, purposive sampling selects participants based on specific characteristics to enhance the understanding of theories and concepts (Battaglia, 2008; Etikan et al., 2016; Patton, 2015).

In this study, all recommended participants were first confirmed according to the inclusion/exclusion criteria. Of those eligible, the researcher then considered the maximum number of participants at the minimum number of locations (given logistics of the researcher's time and finances) while still maintaining a diverse sample. Given the number of children available, participants could not be selected at random. Children were only confirmed for participation if they met all the inclusion/exclusion criteria, and parent/guardian consent and child assent were obtained.

Inclusion/Exclusion Criteria

The inclusion/exclusion criteria are defined below, including a rationale for each. Inclusion/exclusion criteria ensured a communicatively homogeneous sample to address the research questions. Participating children were required to:

- Have a hearing loss (mild to profound; unilateral or bilateral), identified in an IEP as the primary disability, having goals addressing the impact of hearing loss (academic or social/emotional) upon their education. This criterion ensured participants represented the target population.
- Use listening and spoken English as the primary method of communication. This criterion ensured children were able to access, and potentially comprehend, the spoken-English story problems.
- Have no identified specific communication or developmental disabilities (e.g., dyslexia, intellectual disability, autism) that interfere with academic learning. This criterion ensured participant's access and potential comprehension of the spoken-English story problems were not limited or confounded by additional factors.
- Be in kindergarten, Grade 1, Grade 2, or Grade 3. This was because the theoretical framework used to construct the story problem-solving tasks and

analyze participant's story problem-solving was grounded in research from children in kindergarten through third grade.

Data Collection

This section outlines the data collection instruments, environment, and procedures for the study.

Instruments

The study used screening instruments and data collection instruments. Screening instruments included parts of a background questionnaire completed by each participant's classroom teacher, the Ling 6 Sound Test, and three counting tasks utilized in Ansell and Pagliaro's studies conducted with signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). Data collection instruments included parts of the background questionnaire, an English listening comprehension test (Clinical Evaluation of Language Fundamentals – 5; Wiig et al., 2013), and two similar sets of nine story problems, differing only by included numbers. This section describes the selection and rationale for each screening and data collection instrument.

Background Information Questionnaire. Each potential participant's teacher completed the background questionnaire after consent and assent forms were signed so the researcher could ascertain whether the child met all the inclusion/exclusion criteria. The teacher was asked to complete the questionnaire, as it gathers information readily accessible to the teacher (i.e., the child's current mathematics grade level, hearing levels, IEP goals, etc.). The questionnaire collected the following information about each child:

- child’s name
- child’s date of birth
- child’s grade level
- child’s current mathematics grade level
- listening age (number of years the child has used an assistive listening device (ALD) such as a hearing aid or cochlear implant)
- hearing level unaided in the left and the right ears (i.e., mild, severe, profound, etc.)
- which assistive listening device(s) the child uses (i.e., hearing aid only, hearing aid plus FM, etc.)
- any identified disabilities (not including hearing loss)
- languages or communication systems the child uses (other than spoken English)

This information allowed the researcher to assess the potential participant against the inclusion/exclusion criteria. After this was confirmed, the participant’s reported date of birth identified the starting item on the language screening assessment record form while hearing levels and assistive listening device (ALD) use informed the room set up (e.g., if a child heard better in one ear than the other, the researcher took this into account when sitting concerning the participant).

Ling 6 Sound Test. The Ling 6 Sound Test tested each participant’s sound (phoneme) detection via their assistive listening device. Data confirmed that each child’s auditory equipment was working, ensuring participants had the best possible opportunity to auditorily access, and thus comprehend the researcher's instructions and story problem videos. This test uses six sounds that, together, make up the acoustical breadth of the English language in a single phoneme: aaaahhh ("b**aaa**"), eeeee, ("m**e**"), oooo ("t**oo**"), shhh ("**sh**ip"), sssss ("cup**s**"), mmmm ("**m**uffin"). The researcher stood 1 meter behind the child and said a sound; the child then raised his/her hand. If the child raised a hand for all six sounds, their auditory equipment was assumed to be in optimal working condition. If the child failed to raise a hand for a sound, the researcher moved closer to the child (0.5 meters) and repeated that sound. All children passed the Ling 6 sound test at a distance of 1 meter. Multiple studies have concluded that the Ling 6 sound test is an appropriate screening measure of optimal ALD functioning for children based on audiometric measurements (Glista et al., 2014; Scollie et al., 2012) and surveys of audiologists regarding their practice (Glista et al., 2014).

Three Counting Tasks. To determine which version of the story problems (A or B; see *Story Problems* section below) was most appropriate for each child, the researcher asked each child to perform three counting tasks. First, the child counted up from 1 as high as possible, stopping at 30. The researcher then asked the child to "count on from 14 (or from 4 if the child could only count to 10)," to test whether the child could count on from a given number or whether they must start from one again. Finally, the researcher showed the child eight blocks and asked the child to count them. The blocks were then covered up with a sheet of paper, and the researcher asked the child how many blocks were covered up. If the child responded "eight" without having to re-count the set, it was assumed the child understood cardinality (i.e., that a number represents the total set of objects). If the child had to recount all the cubes, the child was assumed to have not yet reached this understanding. If the child counted to at least 20 accurately and fluidly, counted on accurately and fluidly from 14, and displayed cardinal understanding, Set A (numbers 1 to 20) was given. If the child struggled to count accurately and fluidly up to 20, was unable to count on from 14, or did not display cardinal understanding, Set B (numbers 1 to 10) were given.

Clinical Evaluation of Language Fundamentals-5 (CELF-5; Wiig et al., 2013). This instrument was selected for three reasons. First, as the testing manual states, this assessment is valid for children with hearing loss (Wiig et al., 2013). Second, the subtests within the CELF-5 (Wiig et al., 2013) are statistically designed to be independent of each other; that is, each subtest can be administered and scored independently of any other subtest within the CELF-5 (Wiig et al., 2013). This created an opportunity for language screening while reducing the child's out-of-class time, as opposed to a full language assessment. Third, the CELF (version not reported) is widely used and accepted in the field of deaf education as indicated by survey responses from teachers of the deaf/hard of hearing and speech-language pathologists regarding their assessment practices (Bennett et al., 2014; Luckner & Bowen, 2006). It is important to note that these findings may reflect that the CELF is more widely available, not more widely preferred. Regardless, this assessment is familiar to stakeholders in deaf education, which further supported its use in the study.

The CELF-5 (Wiig et al., 2013) was constructed for children 5 through 21 years old and was based on a norming sample of over 3,000 hearing children across the United

States. This test has five target uses of language: (a) Core Language, (b) Receptive Language, (c) Expressive Language, (d) Language Content, and (e) Language Structure. These indexes are broken down into three age ranges: 5-8.11, 9.0-12.11, and 13.0-21.11. Within the CELF-5 (Wiig et al., 2013), the subtest Sentence Comprehension assesses receptive language at the 5.0 to 8.11-year-old age range. This subtest is an appropriate measure for the study for three reasons. First, the Sentence Comprehension subtest requires comprehending sentences of varying lengths, similar to comprehending a story problem made up of two or three sentences of varying lengths. The similarity between a language assessment task and the target task increases the validity and reliability of language assessment scores (Bachman & Palmer, 2010). Second, this measure is constructed for children between 5.0 to 8.11 years of age, allowing the researcher to identify the English language comprehension skills of all participants from kindergarten to Grade 3 (ages 5.0 to 8.11).

In the Sentence Comprehension subtest, the examiner reads a sentence to the child, and the child then points to one of four pictures that best represents the idea(s) within that sentence. The basal (starting) item was calculated from the child's chronological age (reported in the teacher background questionnaire), and four consecutive incorrect responses determined the ceiling. The subtest contains three trial items. Wiig et al. (2013) suggest this subtest will take 5-7 minutes to complete, with an average of 6 minutes.

Using the CELF-5 (Wiig et al., 2013) to gather data regarding English listening comprehension was more appropriate than using the information within school records because one person performed the assessment from one common measure within a short time frame; such consistency increased measurement reliability and limited variability related to factors of child development, scores from multiple instruments, and scores from multiple assessment administrators.

Story Problems. Nine story problems were presented to each child in spoken English on video; the comparable problems are provided in written form in Table 2. They are designed to elicit children's story problem-solving strategies and define relative story problem difficulty. There are two versions of each story problem, identical in all aspects but numbers (set B is in parentheses). These ensured the numbers within the story problems were appropriate to elicit the typical strategies a child would use to solve a given story problem type.

Table 2

Story Problem Types for Study

Problem Type	**Position of Unknown Quantity**		
Join	**Result Unknown** Aaron had 3(2) cars. Jumal gave him 8(6) more cars. How many cars does Aaron have altogether?	**Change Unknown** Bob wants 15(8) worms. He found 9(5) already. How many more worms does he need to find?	**Start Unknown**
Separate	**Result Unknown** There were 11(6) children on the playground. 7(4) children went home. How many children were still playing on the playground?	**Change Unknown**	**Start Unknown**

Table 2

Cont.

Problem Type	**Position of Unknown Quantity**		
Part/Whole	**Whole unknown** There are 4(2) girls and 9(7) boys playing soccer. How many children are playing soccer?	**Part unknown** Megan has 13(9) balloons. 8(6) are red and the rest are blue. How many blue balloons does Megan have?	
Compare	**Difference Unknown** Rachael built a tower 8(4) blocks high. Pat built a tower 14(7) blocks high. How much higher is Pat's tower than Rachael's?	**Compare Quantity Unknown**	**Referent Unknown**
Grouping	**Multiplication** Kelly has 3 bags of candy. There are 4(2) candies in each bag. How many candies does Kelly have?	**Partitive Division** Jake has 12(6) cookies to sell. He put the cookies into 4(2) bags with the same number of cookies in each bag. How many cookies were in each bag?	**Measurement Division** Paul has 15(8) caterpillars. He put 3(2) caterpillars in each jar. How many jars did he put caterpillars in?

Note. Framework from Carpenter et al., 2015; Story problem content from Ansell and Pagliaro (2006), and Pagliaro and Ansell (2012).

Only nine of the 14 possible one-step arithmetic story problem types were included, as research suggests kindergarten children remain engaged for about ten to twelve minutes on an independent academic task before becoming fatigued or distracted (Davenport, 2013; Ponitz et al., 2009). Carpenter et al. (2015) video-recorded hearing children solving the same story problem types as in the present study; using the time length of these videos as a guide, it was estimated that completing one story problem-solving task within this study would take 1-2 minutes. However, these videos were taken of children accustomed to solving story problems daily; some oral d/hh participants took longer. It was felt more than nine story problems could increase the total time spent story problem-solving beyond the recommended 10-12 minutes, possibly introducing fatigue or distraction as a confounding variable into the results. Additionally, past studies with children in kindergarten to Grade 3 using the CGI conceptual frameworks also used less than 14 story problems. Researchers in these studies stated kindergartener’s stamina and focus were a deciding factor in presenting a limited number of story problems (Ansell & Pagliaro, 2006; Carpenter, 1985; Carpenter & Moser, 1984; Carpenter et al., 1993).

The nine story problems were taken from research regarding the strategy use and relative story problem difficulty in signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). Additionally, the story problems were presented in an order identical to Ansell and Pagliaro’s work (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012), as follows: (a) Join Result Unknown, (b) Separate Result Unknown, (c) Part/Whole - Part Unknown, (d) Part/Whole - Whole Unknown, (e) Compare Difference Unknown, (f) Join Change Unknown, (g) Multiplication, (h) Partitive Division, and (i) Measurement Division. Using the same story problems, presented in the same order, allowed for stronger comparisons between oral d/hh and signing d/hh children.

All nine story problems were presented to each participant via videos on the researcher’s computer. Each video depicted the same man, in the same setting, reciting each story problem in spoken English. Before data collection, these videos were shown to three oral d/hh children not involved in the study in any way, and their feedback indicated the video sound and visual quality was clear. These videos provided simultaneous visual (i.e., speechreading) and auditory access to the story problems. Such dual-sensory access is critical as many d/hh children rely on speechreading to fully comprehend spoken language, using what they see to “fill in” what they cannot hear (Kyle et al., 2016).

To maximize visual access, the computer measured 21.2 inches diagonally, larger than the 4.3-inch minimum screen size recommended for effective video comprehension (Maniar et al., 2008; Raptis et al., 2013). The speaker on the video spoke against a plain unlit background, ensuring their face was clearly visible and not cast in shadow, reducing barriers for participants who rely on speechreading.

The audible portion of the videos was presented in one of two ways, depending on whether the participant used an FM system or not. (An FM system is a personal relay system, similar to a walkie talkie, in which a transmitter picks up sound from the environment and sends it to a receiver connected to the child’s assistive listening device, giving the child direct access to sound without background noise.) If the participant used an FM system, it was connected to the computer through a cord plugged into the headphone jack on the laptop, sending audio from the laptop directly to the child’s FM system. If the participant did not use an FM system, auditory access was provided through wide-frequency response speakers placed on either side of the computer. Wide-frequency response speakers were used because they project the low-, mid-, and high-pitched sounds of the English language at a consistent volume level; in contrast, built-in laptop speakers typically project the low- and high-pitched sounds more quietly than the mid-pitched sounds, distorting speech (Dicomo, 2005; Militano, 2011). This difference in technology use was appropriate because it maximized auditory access for each participant.

Environment

The following section describes the arrangement of the environment for data collection. These tasks ensured optimal access to the researcher's instructions and the story problem content.

Room. Because the participants and the researcher had some form of hearing loss, and the researcher also has some vision loss (legally blind in the right eye), the arrangement of the environment was critical to ensure effective communication between both parties. All spaces for data collection were quiet and well-lit. The researcher and participant sat at a table together, facing the camera. The participant's chair was placed approximately 1 meter away from the speakers and computer, the distance recommended by the American National Standards for creating optimal listening conditions (Champlin & Letowski, 2014). All participants wore the researcher's FM transmitter, ensuring the researcher had optimal access to the participants' statements.

Noise Level. The average noise level within the room was 65 dB(A) or lower. Multiple studies of classroom noise levels suggest the average noise level is 65 dB(A) (Jamieson et al., 2004; Shield et al., 2015). The noise level within the data collection rooms was monitored using the iPhone app NIOSH Sound Meter (Centers for Disease Control and Prevention, 2016). This app was selected because it has been featured in multiple studies of iPhone sound meter apps (Chucri et al., 2014; Kardous & Shaw, 2014;

Roberts et al., 2016), suggesting a certain level of acceptability within the field. It also has an external website to support user interpretation of the recordings. Researchers have compared the measurements of machines professionally calibrated to measure environmental noise to the measurements of iPhone apps designed to measure environmental sound and have concluded that iPhone apps are acceptable for a general measure of environmental noise (Kardous & Shaw, 2014; Murphy & King, 2016). The researcher took a measured reading at the start and end of each session and periodically glanced at the screen during the session to monitor the noise level. Noise level recordings were never above 65 d(PA) in any sessions.

Video Cameras. Each session was recorded with two video cameras. The video cameras were arranged so that one recorded the participant and his/her work area, and the other recorded the full interactions between the researcher and participant. This camera arrangement captured all data for later coding confirmation and inter-rater reliability. It also served as another check for any noise or visual interruptions of which the researcher was unaware during the live taping.

Materials. Each child had the following materials available within reach on the table: a large (11 x 17) pad of paper, a black and a green marker, and 20 black and 20 green Unifix cubes. The laptop was placed in front of the child, with the speakers on either side. The researcher also had an envelope containing the coding sheets, the English comprehension measure response booklet, and a notebook for anecdotal (field) notes or drawings. Extra pads of paper and a box of additional markers and (researcher) hearing aid batteries were also on the table.

Preview Videos. Before viewing the first story problem video, the speaker volume and laptop screen angle were set at an optimal level for each child. The child was shown two preview videos, identical in formatting to the story problem videos, featuring the same man in the same setting with the same lighting. In these two videos, the man said either "The quick brown fox jumped over the lazy dog" or "I like butter on my toast for breakfast." The child indicated if the speaker volume was too quiet/loud or if the screen angle needed adjusting.

Procedures

The data collection procedures included two parts: a subtest of the Clinical Evaluation of Language Fundamentals – 5 (Wiig et al., 2013) and the story problem-solving tasks, the latter of which mirrored those of Carpenter et al.'s (2015) studies with hearing children and Ansell and Pagliaro's studies with signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012), allowing comparisons of results.

Clinical Evaluation of Language Fundamentals – 5 (Wiig et al., 2013). The researcher gave the Sentence Comprehension subtest of the Clinical Evaluation of Language Fundamentals – 5 (Wiig et al., 2013) using procedures as outlined in the testing manual. No accommodations regarding test content or presentation were needed beyond using the participant's FM system if they had one. The researcher is formally certified to conduct and score Level B assessments. Results were used to divide the data into groups of d/hh children with age-appropriate listening English comprehension and below-age-appropriate listening English comprehension to answer Research Questions 3 and 4 during coding.

Counting Tasks. Each child completed the three counting tasks, and from the results, the researcher selected whether to show story problem set A or B.

Story Problem-Solving Tasks. To open, the researcher told the child that they would watch a man tell a story on video with a question at the end and that their task was to answer that question. The child was told they could watch the video as many times as needed. The child was then shown the materials with the following explanation: “You can use any of these materials any way you like to help you answer the question at the end of the story. You can also use your fingers if you like, or just think.” When the child indicated that the directions were understood, the first story problem video was shown. During this process, the researcher acted as an outside observer, not interjecting with thoughts or suggestions while the child worked. When the child provided an answer, and if the strategy used was clear, then the researcher responded, “Great! Are you ready for the next story?” If the strategy used by the child was unclear, the researcher asked the child to explain their thinking with a statement such as, “How did you know that?” or “Can you tell me what you were thinking?” Such interview questions helped the researcher understand the child’s strategy selection and thinking.

As suggested by research on interview methodology, the researcher engaged in three practices to maximize the richness of interview data. First, the researcher sought to understand rather than be understood, actively attempting to view the task (story problem-solving) from the interviewee’s (child’s) point of view by suspending disbelief or opinions about the subject (Fontana & Frey, 1994; Ginsburg, 1997). Second, the researcher attended to a child’s words *and* actions, such as how fingers are used, what parts of the story problem are attended to, and facial expression and gesture—these actions can inform an adult's understanding of a child's mathematical thinking (Ginsburg, 1997). Third, knowing a child may be more likely to talk when the researcher was not talking (because when one is talking they are not listening), the researcher did not ask leading questions (i.e., "So, were you thinking _____ when you solved this?"), and sought to understand rather than correct (i.e., responding "Nice work!" or "Good job!" instead of "You got the answer right!"; Ginsburg, 1997). Once the researcher was clear on the child's strategy, or if the child was unable to explain their thinking, the researcher progressed to the next problem, again saying, "Great! Are you ready for the next story?" This phrase, said whether or not the child gave a correct answer, provided feedback without implying any "success" or "failure" of the child's work and without directing the child toward or away from a particular solution or strategy.

Periodically, the researcher reminded the child that using cubes, fingers, or just thinking were perfectly acceptable ways to try and answer the question at the end of the story problem, and that they could request to watch/listen to the story problem again if they wanted. For all participants but one (due to not responding to the first four story problems), this process was repeated nine times, once for each of the nine story problems. After each story problem was complete, any writings/drawings the child made are set aside and marked for later data processing.

Data Coding

The CELF-5 (Wiig et al., 2013) scores were coded as above, at, or below age level. The CELF-5 Testing Manual recommends scaled scores be used for assessment when only one subtest is used (Wiig et al., 2013). Scaled scores were calculated using the raw score and the participant's chronological age. Using the classifications in Table 3, scores of 7 or higher were coded as age-appropriate spoken English comprehension, while scaled scores of 6 or lower were coded as below-age-appropriate spoken English comprehension.

Table 3

Scaled Scores Descriptors from the CELF-5 Testing Manual (Wiig et al., 2013)

Scaled Score	**About the Mean**	**CELF-5 Descriptor**	**Code in Present Study**
13 and above	+1 SD and above	Above Average	Age-appropriate
8 to 12	+ or - 1 SD	Average	Age-appropriate
7	-1 SD and below	At risk	Age-appropriate
6 and below	Below -1 SD	Low to Very low	Below-age-appropriate

Video recording data were coded for strategy use and relative story problem difficulty. The codebook (see Appendix B) defines the coding system for the study. This same coding system was utilized in parallel studies with signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012) and hearing children (i.e., Carpenter & Moser, 1984). Each story problem was coded for story problem type (e.g., Join Result Unknown, Part/Whole – Part Unknown, etc.), having a correct or incorrect answer, solution strategy (e.g., modeling, counting, or fact-based), and strategy viability (i.e., could reach the correct answer or not). Additionally, whether the child attempted the story problem or not and whether that child's strategy use was clear or unclear to the researcher was coded. If a child switched strategies partway through solving the story problem, or if the child wished to solve the story problem again in a different way, the final strategy was the only strategy coded. Coding occurred during data collection at the end of each day and was confirmed after data collection based on the video recordings. All coding, both during data collection and during a review of the videos, was recorded on the same coding sheet. Final codes were entered into SPSS for analysis.

Inter-Rater Agreement

A measure of inter-rater agreement ensured the reliability of data coding. As Gwet (2012) states in his *Handbook of Inter-Rater Reliability 3rd Edition*, calculating the inter-rater agreement of data coding is critical to ensure findings do not reflect researcher error. For the present study, the researcher trained one inter-rater on the nine different story problem types and the three different strategy types—modeling, counting, and facts. The video examples used to train the inter-rater were drawn from Carpenter et al. (2015), and inter-rater coding commenced after the inter-rater accurately coded each type of strategy independently twice; two rounds of training were necessary to achieve this. No methodological literature describing how much data an inter-rater should code (i.e., 20%, 50%, 100% of the data) could be identified; however, the field tends to have inter-raters code at least 20% of the data until agreement is reached.

The inter-rater reliability level was calculated in Excel using Kappa (Hallgren, 2012). Multiple researchers have suggested Kappa is superior to a simple percentage (number of times researcher and inter-rater agree ÷ total) when calculating inter-rater agreement because Kappa statistically accounts for the possibility that the researcher and inter-rater both incorrectly coded some items (Fleiss, 1971; Hallgren, 2012; McHugh, 2012). Additionally, Hallgren (2012) and Fleiss (1971) suggest Kappa is particularly well-suited to calculate the inter-rater agreement of nominal data; that is, data split into categories. The formula used to calculate Kappa is as follows (Hallgren, 2012):

$$K = \frac{P(a) - P(e)}{1 - P(e)},$$

where P(a) is the percent of agreement, calculated by dividing the number of times the researcher and inter-rater agreed by the number of story problems viewed. P(e) is the probability of expected agreement, calculated in the following three steps:

1. The number of times the researcher codes each strategy is divided by the total number of story problems viewed; this is repeated for the inter-rater's coding, and these percentages are then multiplied.
2. Each of the researcher's and inter-rater's strategy codes from step one is subtracted from 1.0 to find the number of times a strategy was *not* coded, and these percentages are then multiplied.
3. Adding the probabilities from Steps 1 and 2 together gives P(e).

In this formula, 1 indicates perfect agreement, 0 indicates random agreement, and -1 indicates perfect disagreement. The minimum acceptable Kappa level for inter-rater agreement was 0.91, where 82-100% agreement is considered reliable (McHugh, 2012).

Given the above, 20% of the recorded story problem-solving attempts (n = 42) were randomly selected for coding by first assigning a unique number to each story problem and then using the RANDBETWEEN function in Excel to list 42 story problem numbers randomly. The initial round of inter-rater coding had a Kappa value of -0.78, not meeting the minimally acceptable rating of 0.91. Upon examining the disagreements and meeting with the inter-rater, the researcher realized the code “unidentifiable strategy” was sometimes used instead of “Non-viable strategy.” This is a critical distinction, as “unidentifiable strategies” were not included in analyses of strategy use, but non-viable strategies were.

The researcher addressed this by double-checked her own codes on all story problems coded “unidentifiable” to see if any were coded incorrectly. As a result, 10 strategies were added to the final dataset: Eight were non-viable, and two were viable fact-based strategies. Additionally, the inter-rater coding sheet was restructured to include additional examples describing the difference between an unidentifiable strategy and a fact-based strategy because most of the disagreements were with fact-based strategies. The second round of inter-rater coding, performed on another set of 42 randomly selected story problem-solving attempts, yielded a Kappa measure of 0.95, meeting the minimally acceptable rating of 0.91, strongly indicating the codes were valid.

Data Analysis

Data analysis was descriptive and quantitative. Statistical analyses included descriptive statistics and Chi-square analyses. The alpha value (significance levels) for Chi-square was set at $p < .05$, following both the minimum level required by the Council for Exceptional Children for research supporting evidence-based practices in special education (Cook et al., 2014) and the significance level used in studies with signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012).

Conclusion

This chapter has outlined the study's methodology. Participants were d/hh children in kindergarten to Grade 3 who used listening/spoken English as their primary means of communication. Data were coded for patterns of strategy use and relative story problem difficulty. An inter-rater reliability level of 0.95 established the study's internal validity. Descriptive statistics and Chi-square analyses identified significant and non-significant patterns regarding story problem correctness, relative story problem difficulty, and patterns of strategy use. The study's sample embodied characteristics similar to comparison studies (Ansell & Pagliaro, 2006; Carpenter et al., 2015; Pagliaro & Ansell, 2012). That is, participants had a hearing loss like those in the Ansell and Pagliaro investigation (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012) but used the language and communication mode of those in Carpenter et al.'s studies (Carpenter et al., 2015). From these similarities, for the first time, oral d/hh children's one-step arithmetic story problem-solving has now been described and placed in context with their signing d/hh and hearing peers.

CHAPTER IV

RESULTS

Introduction

This study describes the story-problem solving of primary-aged deaf/hard of hearing children who primarily use listening/spoken English for communication (oral d/hh children). The design parallels previous investigations with Deaf children who use American Sign Language (ASL) (signing d/hh; Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012) and hearing children who use English (Carpenter et al., 1999, 2015). The oral d/hh children watched nine different types of story problems, taken directly from Ansell and Pagliaro's work (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). See Appendix A for the titles and abbreviations of each story problem type. Findings are drawn from coding the answers participants provided and the types of strategies they used as they attempted to solve these nine story problems. Utilizing the Cognitively Guided Instruction frameworks (CGI; Carpenter et al., 2015), results address the following questions:

1. What patterns of viable and non-viable strategies are used by deaf/hard-of-hearing children, kindergarten through Grade 3, who use listening/spoken English as their primary communication?

2. What is the relative difficulty of one-step arithmetic story problems for deaf/hard-of-hearing children, kindergarten through Grade 3, who use listening/spoken English as their primary communication?
3. How do patterns of strategy use and relative story problem difficulty compare between deaf/hard of hearing children, kindergarten through Grade 3, who use listening/spoken English as their primary communication and have age-appropriate English comprehension or higher, and those who have below-age-appropriate English listening comprehension?

This chapter first summarizes the final sample size and sample demographics and then presents results. Then, oral d/hh participants' patterns of story problem correctness, relative story problem difficulty and distributions of strategy type use are presented. For each topic – correctness, relative story problem difficulty, strategy type use – overall findings are presented and then findings within the following two groups are presented: oral d/hh participants with age-appropriate spoken English comprehension or higher (age-appropriate oral d/hh) and oral d/hh participants with below-age-appropriate spoken English comprehension (below-age-appropriate oral d/hh).

Sample Demographics

Table 4 outlines participant demographics related to three factors: educational setting, language/communication, and use of assistive listening device (ALD). All participants attended one of five schools located in the midwestern, northeastern, and northwestern regions of the United States. Of these five schools, three were separate schools for the deaf, and two were resource room programs housed in public education schools. Across these five educational settings, 75% of the participants (*N* = 18) attended a school for the deaf, and 25% (*N* = 6) attended a mainstream setting with hearing peers. Thus, the sample is skewed toward participants in schools for the deaf.

Table 4

Sample Demographics Summary

Demographic		**Total (*N*=24)**
Educational Setting	School for the deaf	18
	Program in public school	6
Spoken English Comprehension	Age-Appropriate	15
	Below Age-Appropriate	9
Assistive Listening Device Use	CI only (bilateral)	4
	Hearing Aid only (bilateral)	16
	BAHA	1
	CI and Hearing Aid	3
	FM Use	13

Note. CI = Cochlear implant. BAHA = Bone Anchored Hearing Aid. FM = Frequency Modulated (System)

Of all 24 participants, 75% (*N* = 18) were reported by their teachers as working on mathematics at grade level; the remaining 25% (*N* = 6) were reported as working on mathematics content one grade level below their enrolled grade level. No participants were more than one grade level below in mathematics, and none worked above their grade level in mathematics.

All participants were educated at school either through listening/spoken English only (75%, *N* = 18) or through a combination of listening/spoken English and ASL (25%, *N* = 6). As reported by classroom teachers, two participants used languages other than English at home. Based on scores from the subtest *Sentence Comprehension* within the Clinical Evaluation of Language Fundamentals-5 assessment (Wiig et al., 2013), 4% (*N* = 1) of the 24 participants had above-age appropriate spoken English comprehension, 63% (*N* = 15) had age-appropriate spoken English comprehension, and 33% (*N* = 8) had below-age-appropriate spoken English comprehension.

All participants wore at least one assistive listening device. Seventeen percent (*N* = 4) wore bilateral (on both ears) cochlear implants (CI), 12% (*N* = 3) wore a CI and a hearing aid, 66% (*N* = 16) wore bilateral hearing aids, and 4% (*N* = 1) wore a bone-anchored hearing aid (BAHA). No participants wore a unilateral (one ear only) ALD. Additionally, 54% (*N* = 13) of participants wore a frequency modulated (FM) system that further boosted the auditory input from the ALDs they were already wearing. These 13 participants reportedly wore an FM system during instruction; all of these participants used their FM system during all data collection, including the English language comprehension testing, and connected their FM system to the laptop for the presentation of the story problems to maximize access to spoken English. All other participants (*N* = 11, 46%) used their personal ALDs only throughout data collection.

Missing Data

Some of the data from the 24 participants were treated as missing and not included in analyses. Two participants had auditory access concerns for four or fewer story problems, resulting in six strategies being treated as missing data. Additionally, one answer from one participant was treated as missing due to researcher error (the researcher did not hear the child's response; thus, the researcher did not ask subsequent questions). Given these missing data, a total of 209 responses from 24 participants were coded.

All 209 responses were coded for correctness (i.e., correct or incorrect answer), viability (i.e., viable or non-viable strategy), and strategy type (i.e., modeling, counting, fact-based). Of the 209 strategies, 93 were un-codable, and 116 were codable. Un-codable strategies could not be clearly identified as a modeling, counting, or fact-based strategy. For example, if a child answered "six," the researcher would ask, "How did you know that?" and the child may have said, "I used my brain." It is not clear in this context whether the child used a counting or fact-based strategy, so this strategy was coded "un-codable." The remaining 116 codable strategies were coded as one of three strategy types: modeling, counting, or fact-based.

Factors Impacting Story Problem-Solving

The analyses below consider whether spoken English comprehension is significantly linked to story problem-solving correctness, relative story problem difficulty, and strategy use. Within each, overall patterns are first described and examined. Then, statistical links are made between spoken English comprehension levels and story problem correctness, relative story problem difficulty, and strategy use, respectively.

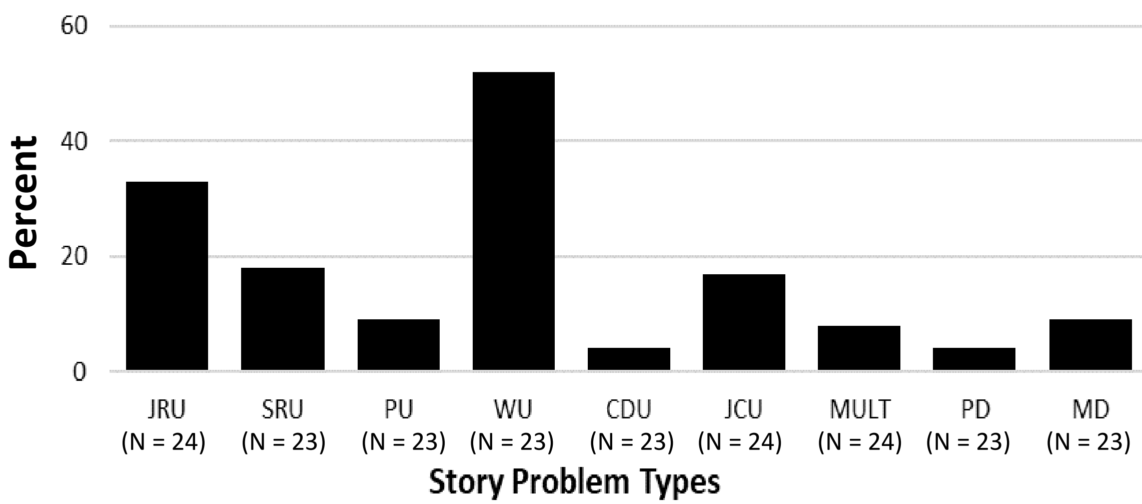
Story Problem Correctness

Overall Correctness

Correctness refers to whether a child's answer was correct or incorrect. Figure 2 shows the percentage of correct and incorrect answers for each of the nine story problem types. The story problem most children answered correctly was the WU story problem type (52%). Two story problems were answered correctly by the least children; these were the CDU (4%) and PD (4%) story problem types. The percentage of correct answers for the remaining six story problems ranged from 6% to 33%.

Figure 2

Percent of Correctness, in Order of Presentation



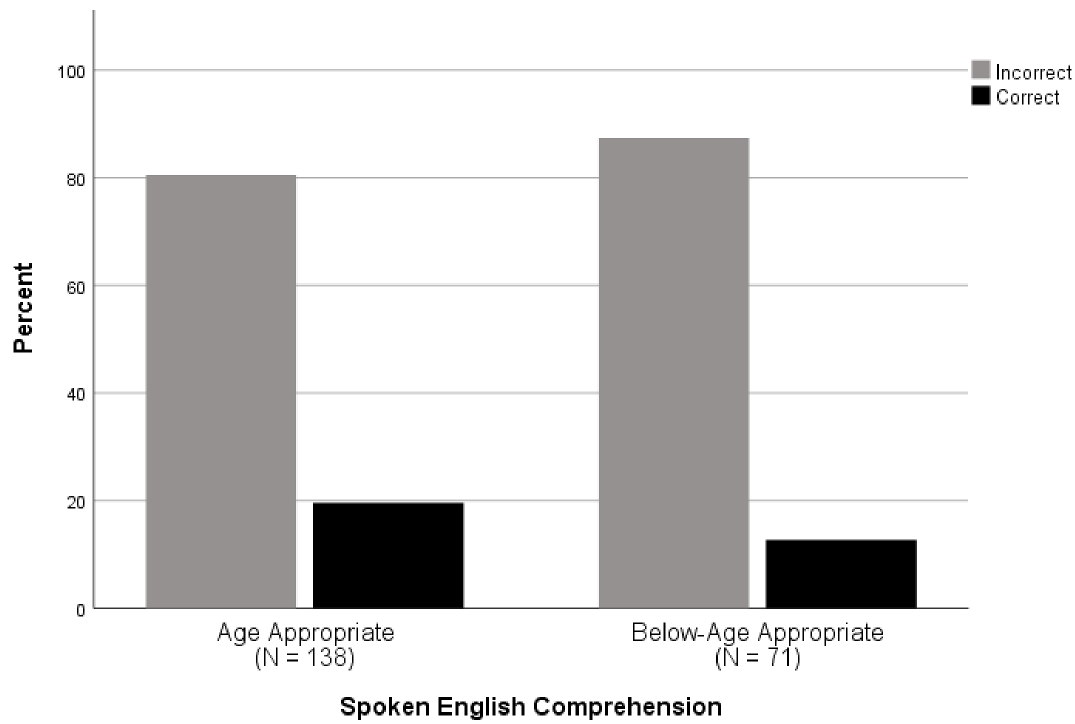
Note. N = number of answers.

Correctness by Spoken English Comprehension

Figure 3 shows the percentages of correct and incorrect answers for children with age-appropriate and below-age-appropriate spoken English comprehension. Both groups provided more incorrect than correct responses. Incorrect responses made up 69% (*N* = 95) to 63% (*N* = 45) of all responses. When comparing these percentages, it is important to recognize children with age-appropriate spoken English comprehension contributed almost twice as many data points as children with below-age-appropriate spoken English comprehension.

Figure 3

Percent of Correctness by Spoken English Comprehension (*N* = 24)



Note. N = number of answers.

To test whether distributions of correct/incorrect answers were significantly different between groups of children based on spoken English comprehension, a Chi-square analysis (2 age/below-age-appropriate spoken English comprehension x 2 correct/incorrect answer) was run. Chi-square compares differences between groups of nominal data (Field, 2009); in this case, correctness and spoken English comprehension. The Phi coefficient was added to account for the binomial nature of the data (Vogt & Johnson, 2011). Significance was set at .05. Results showed no significant differences, indicating spoken English comprehension was not related to oral d/hh children's story problem-solving correctness.

Relative Story Problem Difficulty

Relative story problem difficulty is defined by strategy viability. A viable strategy is one that could lead to the correct answer. For example, the story problem: *There were six children on the playground. Four children went home. How many children were still playing on the playground?* cannot be solved by combining four and six to arrive at a larger quantity such as 10. Thus, such a strategy would be non-viable for this specific story problem type. If a child removes four from six (i.e., 6 – 4 = ?), but gives an incorrect answer of 1, the strategy itself is still viable. Story problems answered more often with viable strategies are said to be relatively easier than story problems answered more often with non-viable strategies. Overall, across all story problem types, the oral d/hh children used a viable strategy 64% (N = 74) of the time and a non-viable strategy 36% (N = 42) of the time.

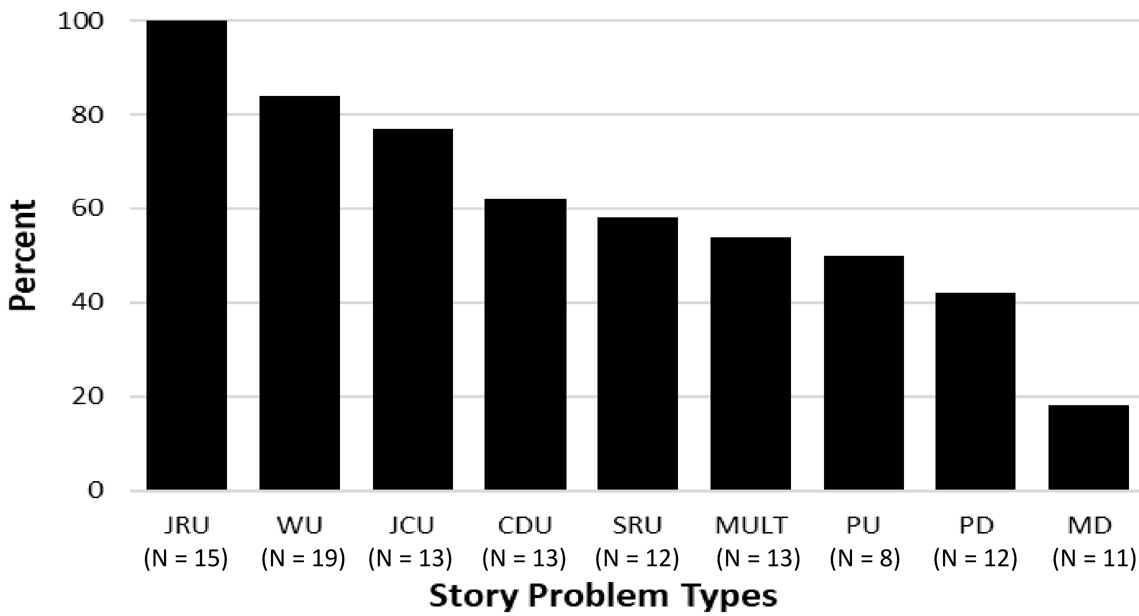
Overall Relative Story Problem Difficulty

Figure 4 shows the distribution of relative story problem difficulty across all nine story problem types. The percentage of viable strategies was greater than 50% on seven of the nine story problems, and greater than 75% for the three easiest story problem types

(JRU, WU, JCU). Percentages of viable strategy use ranged from 100% (JRU) to 20% (MD). The two most difficult story problems produced the largest gap in viable strategy use between all story problem types, from 41% of viable strategies for the PD to 29% of viable strategies for the MD.

Figure 4

Overall Relative Story Problem Difficulty, Easiest to Most Difficult



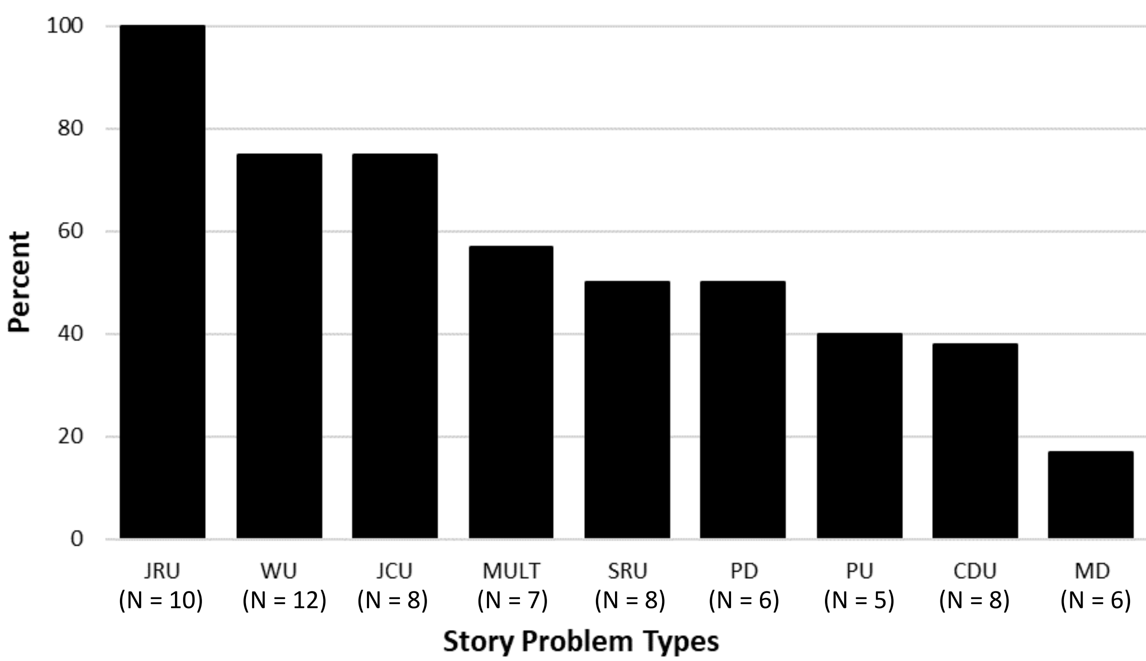
Note. N = codable strategies.

Percentage of Viable/Non-Viable Strategy use by Spoken English Comprehension

Figures 5 and 6 show the distributions of relative story problem difficulty for oral d/hh participants with age- and below-age-appropriate spoken English comprehension, respectively. Both groups found the JRU and WU story problem types to be the easiest and the MD story problem to be the most difficult. As the fifth of nine story problem types, the SRU story problem type was equally easy/difficult for both groups. The most notable difference between the two groups was the relative difficulty of the CDU story problem type. The CDU story problem type was the eighth-most difficult story problem for participants with age-appropriate spoken English comprehension, but the third-most difficult story problem type for participants with below-age-appropriate spoken English comprehension. That is, participants with below-age-appropriate spoken English comprehension found the CDU story problem type relatively easier than did participants with age-appropriate spoken English comprehension.

Figure 5

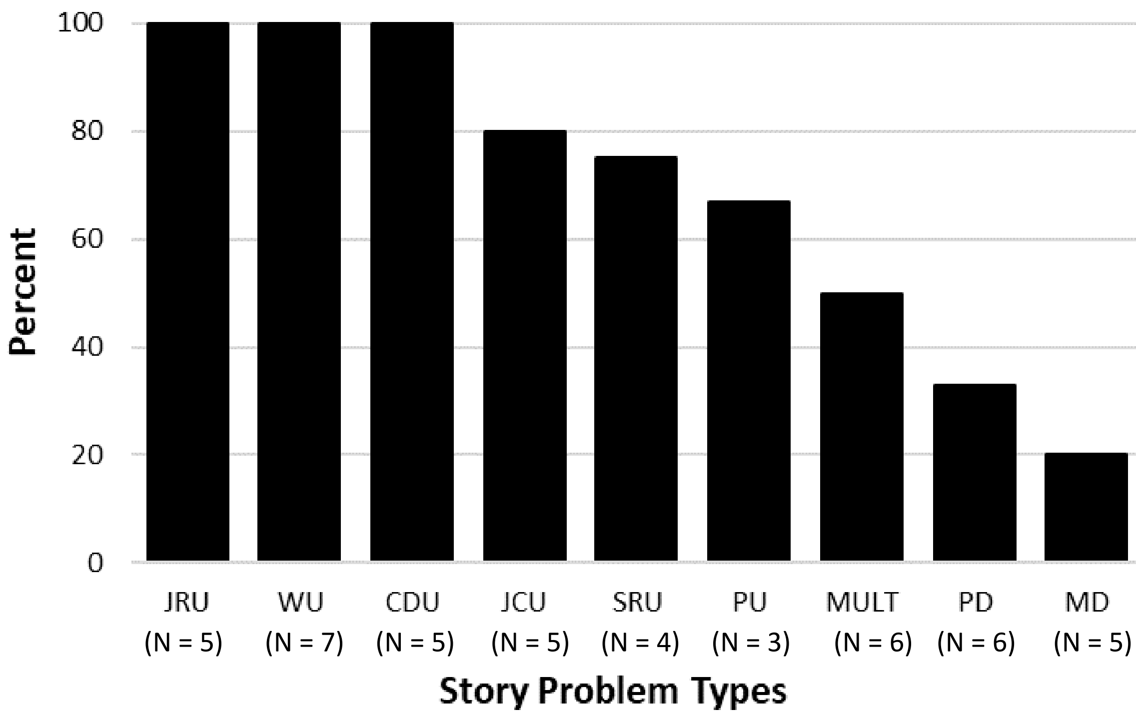
Relative Story Problem Difficulty for Children with Age-Appropriate English Comprehension, Easiest to Difficult (N = 16)



Note. N = Codable strategies.

Figure 6

Relative Story Problem Difficulty of Children with Below-Age-Appropriate English Comprehension, Easiest to Difficult (N = 8)



Note. N = Codable strategies.

To see if spoken English comprehension was significantly related to frequencies of viable and non-viable strategy use, a Chi-square test (2 age-appropriate/below-age-appropriate x 2 viable/non-viable strategy) was run. Chi-square was used because it identifies significant differences between groups of nominal data (Field, 2009), in this case, spoken English comprehension and viable/non-viable strategy use. The Phi Coefficient was added to the analysis as it identifies significant differences in groups of binomial data (Vogt & Johnson, 2011). Significance was set at .05. Results showed no significant differences between frequencies of viable and non-viable strategy use based on spoken English comprehension.

Relationship Between Correctness and Viable Strategy Use

When children use a viable strategy but arrive at an incorrect answer, questions are raised regarding their arithmetic calculation knowledge (Carpenter et al., 2015), but an incorrect answer indicates a calculation error. Conversely, children using a non-viable strategy, but arriving at a correct answer, indicates guessing without comprehending the story problem. Descriptive and statistical analyses were run on the pattern's correctness and viable strategy use to understand whether either factor was present in the dataset.

After removing the 93 un-codable strategies from the correctness variable so that all correct/incorrect codes had a comparative viable/non-viable strategy code, 43% (N = 32) of viable strategies led to a correct answer, while 56% of viable strategies led to an incorrect answer (N = 42).

Given this disparity, a Chi-square test (2 correct/incorrect answer x 2 viable/non-viable strategy) was run to see if there was a significant relationship between overall correctness and viable strategy use. Chi-square was used because it identifies significant differences between groups of nominal data (Field, 2009). The Phi Coefficient was added to the analysis as it identifies significant differences in groups of binomial data (Vogt & Johnson, 2011). Significance was set at .05. A significant difference was found between overall correctness and viable strategy use (χ^2 (1, N =116) = 16.573, p = .000). To explore which story problem(s) might have a significant difference in correctness and strategy viability, Chi-square (2 correct/incorrect x 2 viable/non-viable strategy) with Phi

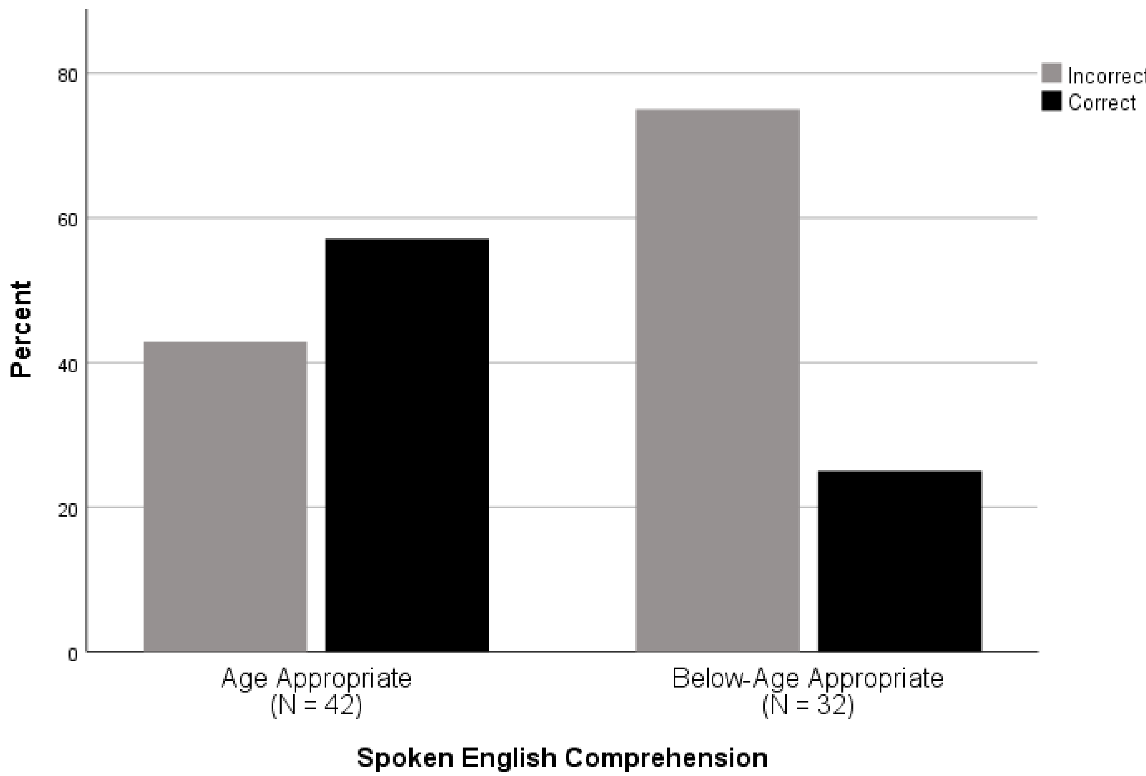
Coefficient tests were run for each story problem type. A significant difference between correctness and viable strategy use was found for two of the nine story problem types: SRU (χ^2 (1, N =12) = 4.286, p = .038), and MD, (χ^2 (1, N =11) = 4.950, p = .026).

Correctness by Viable Strategy Use by Spoken English Comprehension

Figure 7 shows the percentage of correct and incorrect answers that followed a viable strategy from children with age-appropriate and below-age-appropriate spoken English comprehension. To see if there were differences between children with age-appropriate and below-age-appropriate spoken English comprehension based on the frequency of correct answers when a viable strategy was used, a Chi-square analysis (2 age-appropriate/below-age-appropriate spoken English Comprehension x 2 correct/incorrect answers) was run. To ensure these data only reflected patterns of viable strategy use, all non-viable strategies were excluded. Chi-square was chosen as it compares groups of nominal data (Field, 2009), in this case, spoken English comprehension and correct/incorrect answers. The Phi coefficient was used to account for the binomial nature of the data (Vogt & Johnson, 2011). Significance was set at .05. Results showed significant differences between children with age-appropriate and below-age-appropriate spoken English comprehension based on the percentage of correct and incorrect answers following a viable strategy (χ^2 (1, N = 74) = 7.645, p = .006). This suggests the number of times an oral d/hh child's viable strategy led to a correct answer was related, at least in part, to their spoken English comprehension level. Visual inspections of the figure confirm these underlying factors led to oral d/hh children with age-appropriate spoken English comprehension reporting the correct answer (57%) more often than children with below-age-appropriate spoken English comprehension (25%).

Figure 7

Correct and Incorrect Answers from a Viable Strategy by Spoken English Comprehension



Note. N = Viable strategies

Patterns of Strategy Type Use

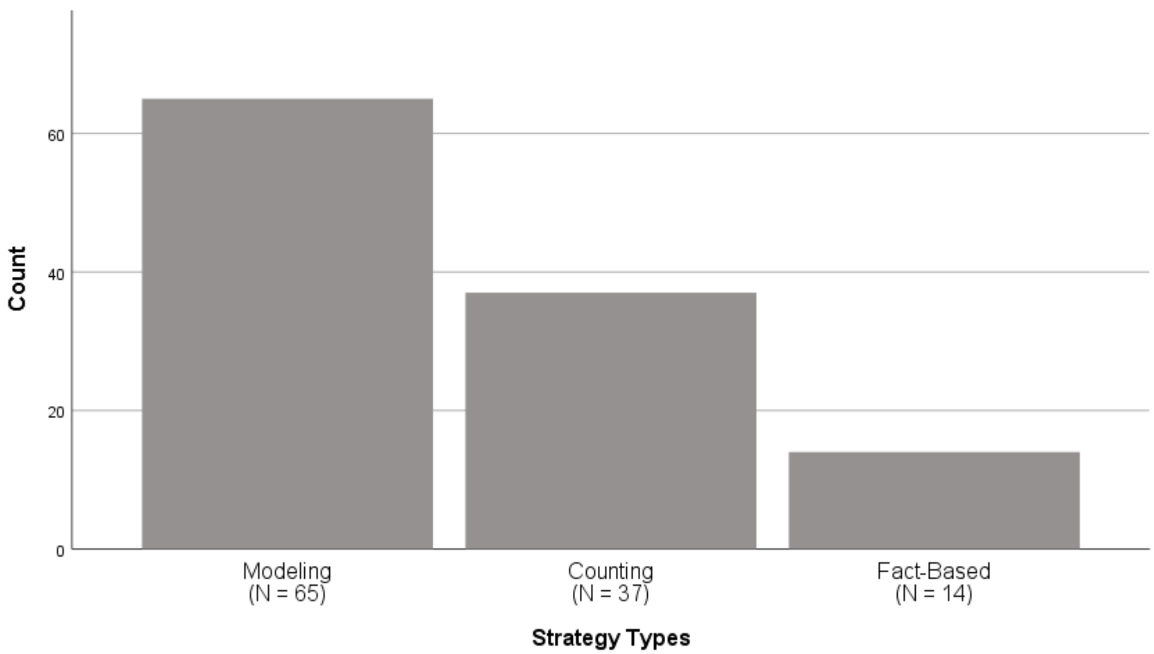
Overall

Strategies were grouped into three types: modeling, counting, and fact-based. Figure 8 shows the overall frequencies of strategy type use (viable and non-viable).

Overall, the oral d/hh participants used modeling strategies most frequently (*N* = 65), counting strategies less frequently (*N* = 37), and fact-based strategies least frequently (*N* = 14).

Figure 8

Frequency of Strategy Type Use (Viable and Non-Viable)



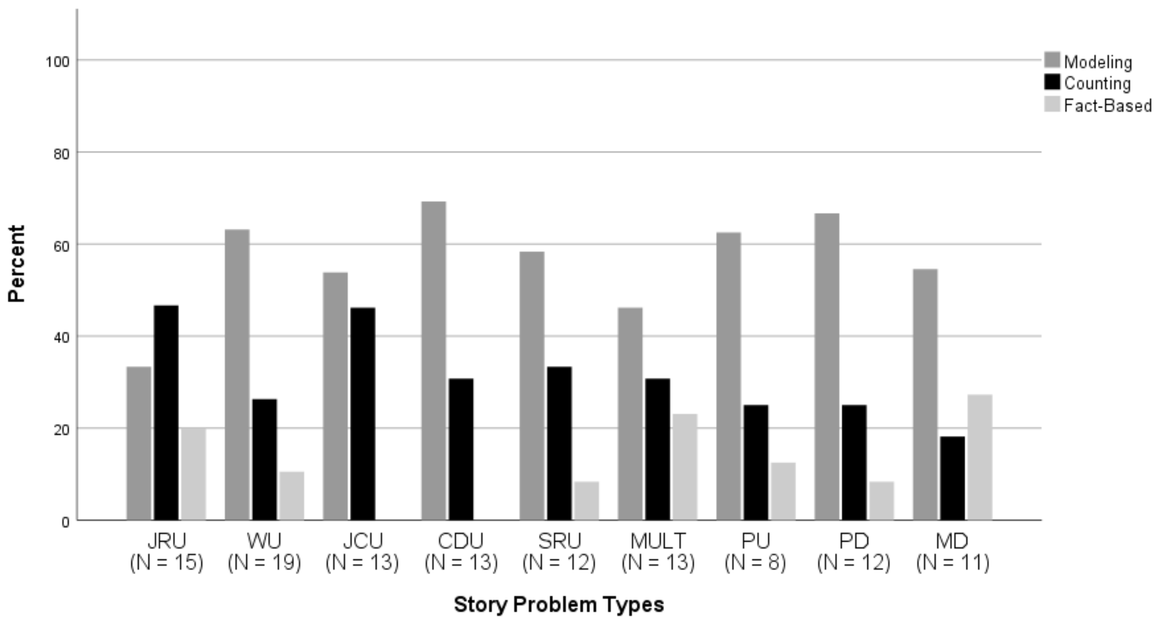
Note. N = Codable strategies.

Strategy Type Use Within Story Problem Types

Figure 9 shows the percentages of overall (viable and non-viable) strategy type used for each story problem type. In the easiest story problem (JRU), counting strategies were used most often, modeling strategies some of the time, and fact-based strategies least often. In the most difficult story problem (MD), modeling strategies were used most often, fact-based strategies used some of the time, and counting strategies used least often. Within the remaining seven story problems, modeling strategies were used most often, counting strategies some of the time, and fact-based strategies least often.

Figure 9

Strategy Type Use by Relative Problem Difficulty (Easiest to Most Difficult)



Note. N = Codable strategies

Strategy Type Use by Language Level

Figures 10 and 11 show the percentage of overall strategy type use for those with age-appropriate and below-age-appropriate spoken English comprehension. Children in both groups used some modeling and some counting strategies to solve all nine story problem types. Oral d/hh participants with age-appropriate spoken English comprehension used at least one fact-based strategy on six of the nine story problem types (67%), while participants with below-age-appropriate spoken English comprehension used at least one fact-based strategy on four of the nine story problem types (45%). Thus, participants with age-appropriate spoken English comprehension used all three strategy types on more story problems.

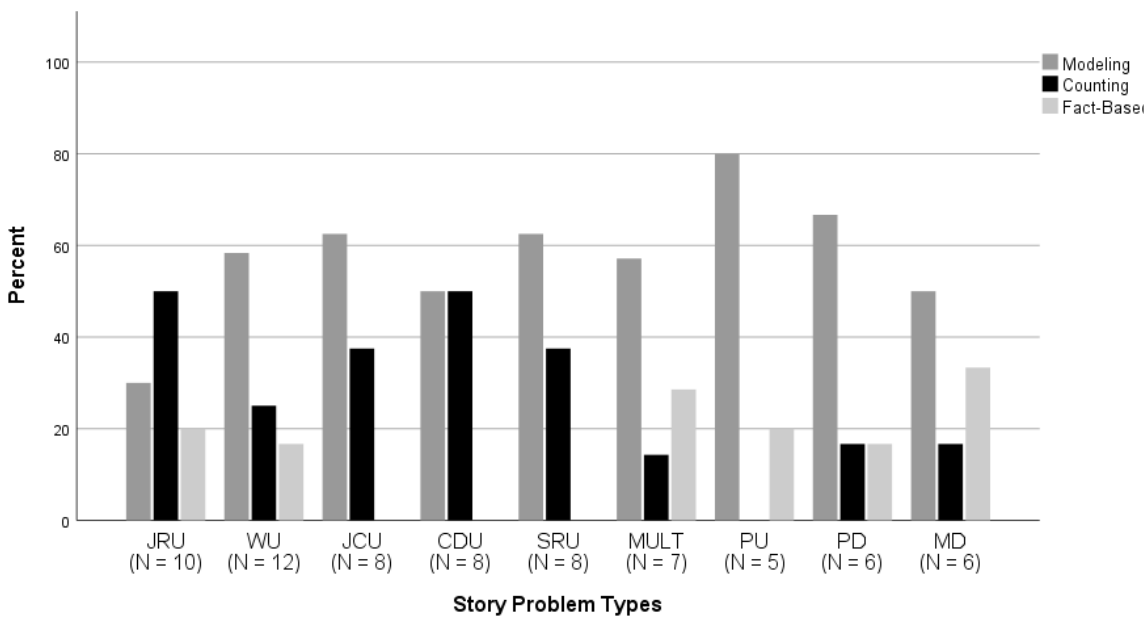
Both groups solved the relatively easiest story problem type (JRU) using an equal number of modeling and counting type strategies, if not more counting strategies. Children with age-appropriate spoken English comprehension solved the remaining eight story problem types predominantly with modeling type strategies. In contrast, children with below-age-appropriate spoken English comprehension solved five of the remaining eight story problem types predominantly using a modeling strategy and three predominantly using a counting strategy. Neither group used fact-based strategies predominantly on any story problem type. While children in both spoken English comprehension groups used modeling strategies predominantly across multiple story problem types, those with below-age-appropriate English comprehension used modeling strategies predominantly across more story types.

On the four most difficult story problems overall (MULT, PU, PD, MD), oral d/hh children with age-appropriate spoken English comprehension used a higher percentage of fact-based strategies than counting strategies, the highest percentage on the most difficult story problem type (MD). In contrast, oral d/hh children with below-age-appropriate spoken English comprehension did not use fact-based strategies more often than counting strategies on any story problem type, though they did use counting and fact-based strategies equally as often for two story problem types.

To understand if spoken English comprehension impacted strategy type use at all, a Chi-square (2 age-appropriate/below-age appropriate language x 3 strategy type use) was run. Chi-square compares groups of nominal data (Field, 2009); in this case, the types of language and types of strategies. The Phi coefficient accounted for the binomial nature of the data (Vogt & Johnson, 2011). Significance was set at .05. Results indicated spoken English comprehension was not related to strategy type use (χ^2 (3, N = 209) = 4.737, p = .192).

Figure 10

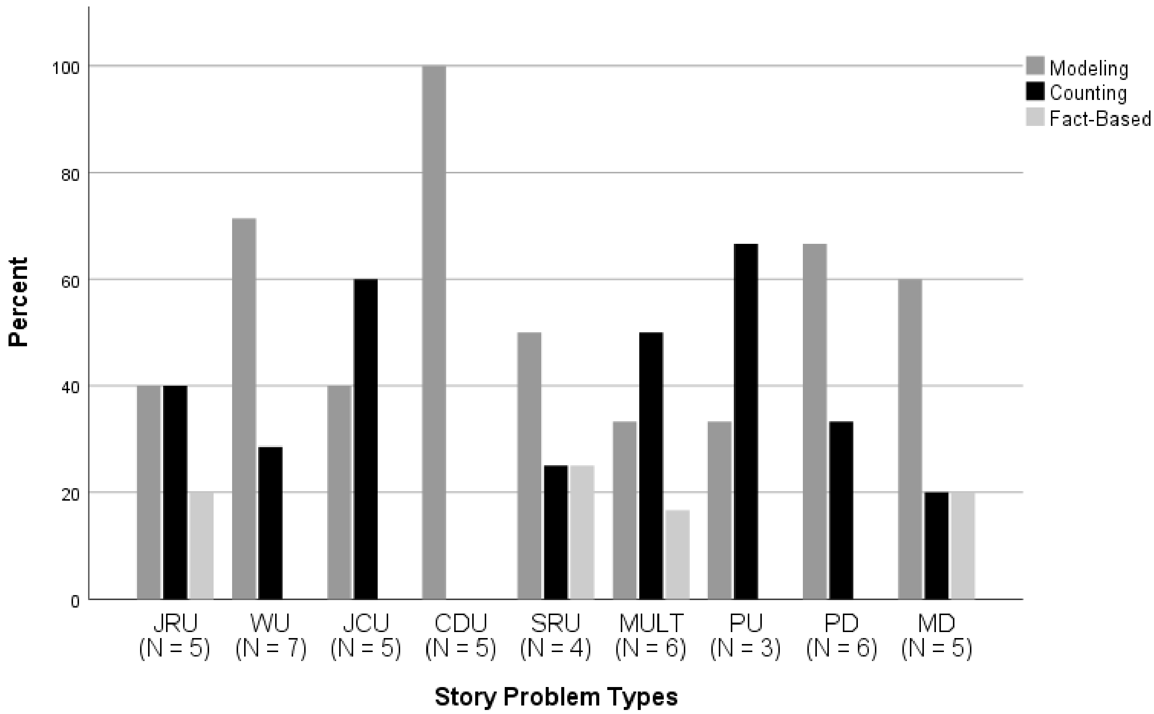
Percentage of Strategy Type Use for Participants with Age-Appropriate English Comprehension, in Order of Overall Relative Story Problem Difficulty (N = 16)



Note. N = Codable strategies

Figure 11

Percentage of Strategy Type Use for Participants with Below-Age-Appropriate English Comprehension, in Order of Overall Relative Story Problem Difficulty (*N* = 8)



Note. N = Codable strategies

Patterns of Viable Strategy Type Use

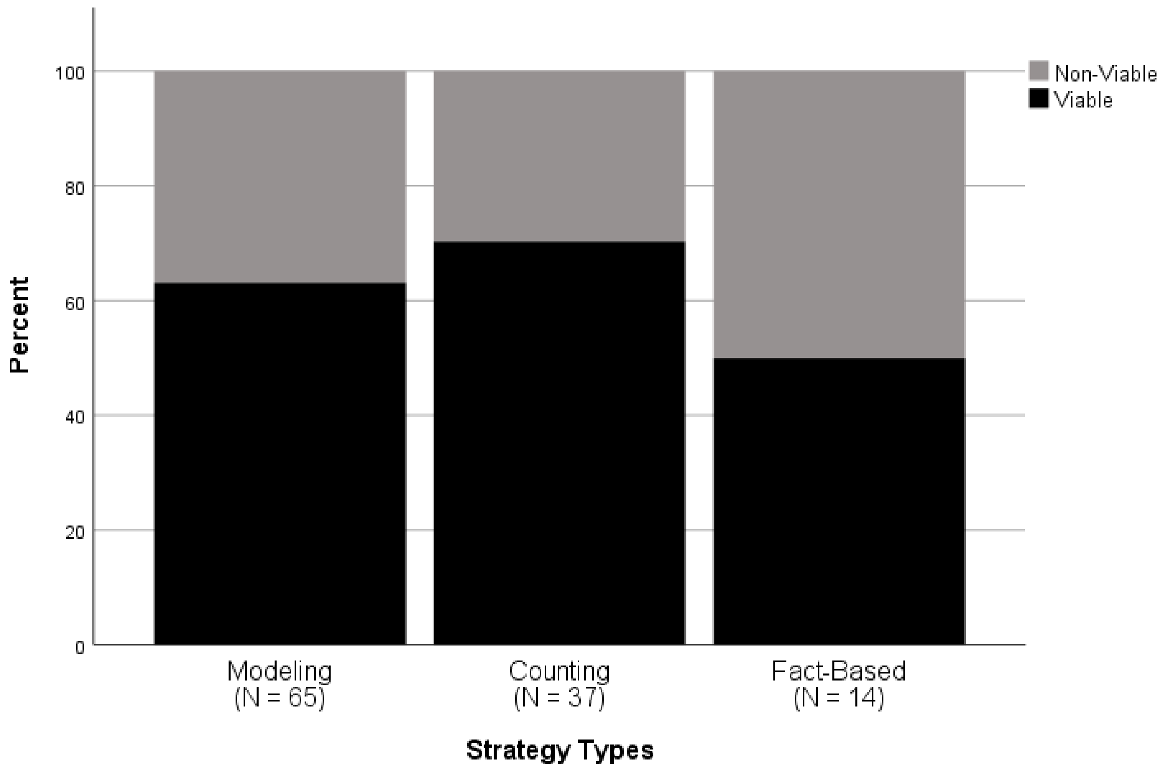
Examining patterns of viable strategy type use can reveal which types of strategies oral d/hh children applied most appropriately. This section first presents overall findings on strategy type use and strategy viability and then presents these same patterns within grade levels and language levels. To conduct these analyses, all non-viable strategies were removed from the dataset before creating the figures and running the statistical analyses.

Overall Strategy Type by Viability

Figure 12 displays strategy types by viability. Of the three strategy types utilized, 63% of the modeling strategies were viable (*N* = 41), 70% of the counting strategies were viable (*N* = 26), and 50% of the fact-based strategies were viable (*N* = 7).

Figure 12

Viable and Non-Viable Strategy Use by Strategy Type



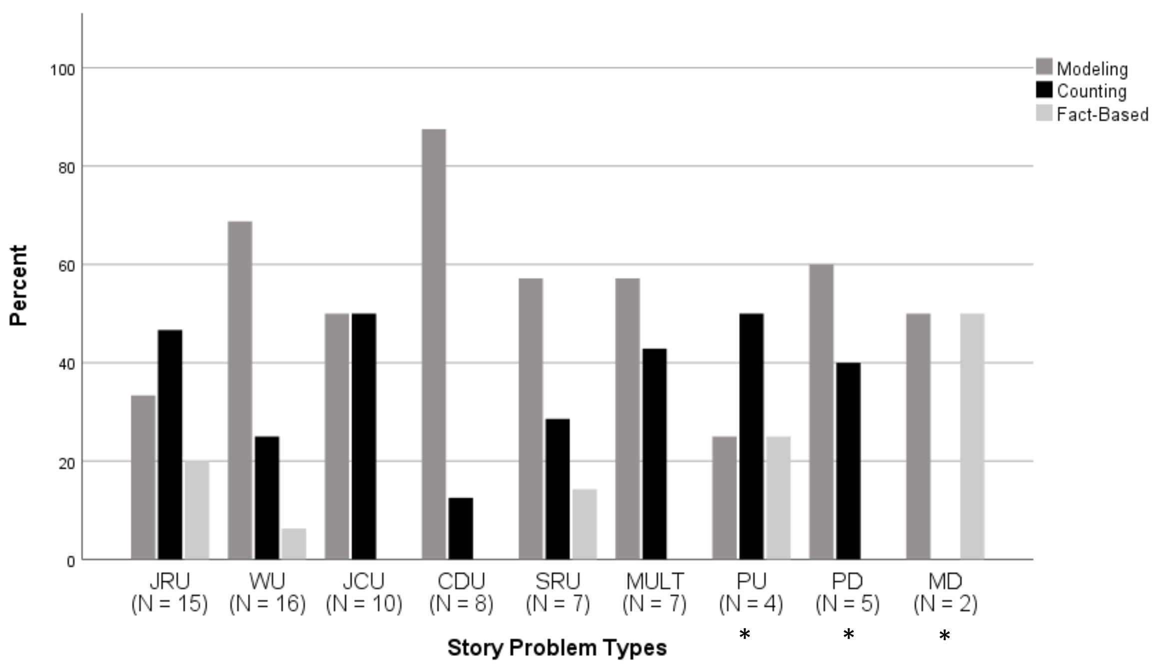
Note. N = Codable strategies.

Viable Strategy Types Across Story Problem Types

Figure 13 shows the percentage of viable strategies used to solve each story problem type. On six of the nine story problems, viable modeling strategies were used most often. On two story problems, viable counting strategies were either used as often or more often than viable modeling and fact-based strategies. On one story problem viable modeling and fact-based strategies were used an equal number of times. Viable fact-based strategies were used least often, being only used on four of the nine (44%) story problems.

Figure 13

Percentage of Viable Strategy Type Use by Story Problem Type, Easiest to Most Difficult



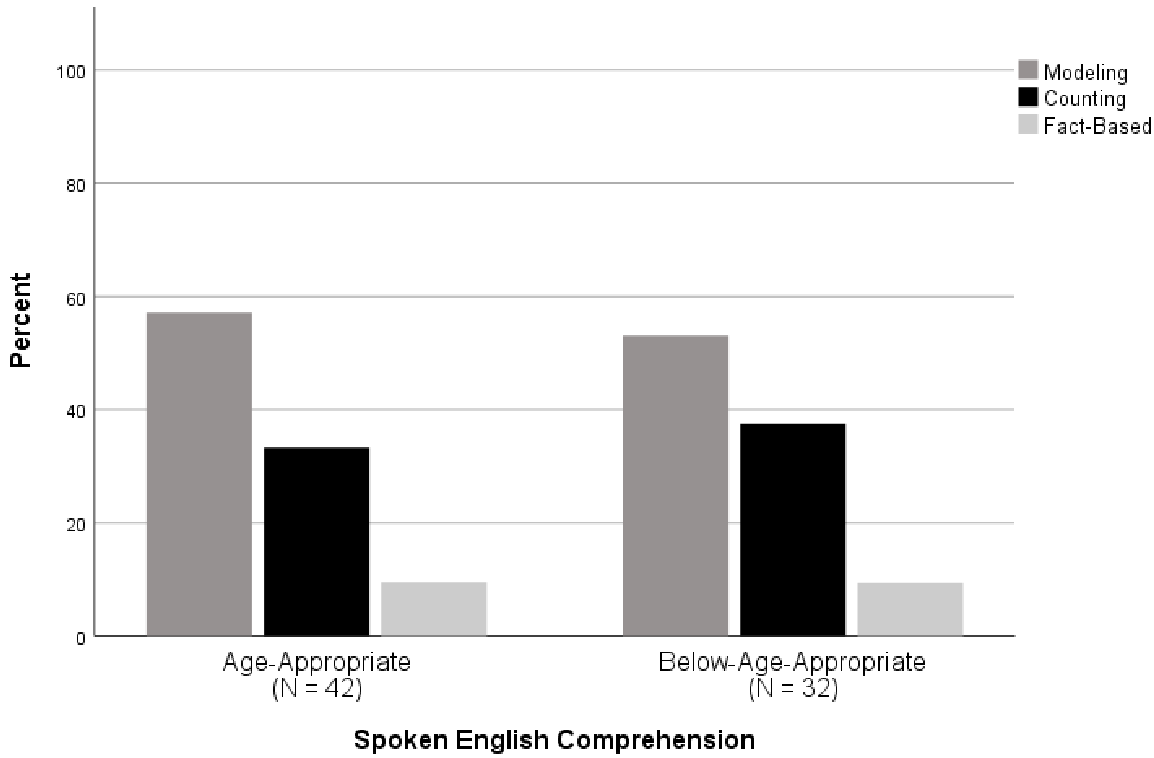
Note. N = number of viable strategies. * = Inflated percentages based on five or fewer data points.

Viable Strategy Types by Spoken English Comprehension

Figure 14 compares the percentages of viable modeling, counting, and fact-based strategies between two groups of children: oral d/hh children with age-appropriate spoken English comprehension, and those with below-age-appropriate spoken English comprehension. Both groups showed similar patterns of viable strategy-type use. Over half of the viable strategies were modeling-type strategies, some were counting-type strategies, and fact-based strategies made up 10% or less of all viable strategies. To test whether any percentages of viable strategy-type use were significantly different between the two groups, a Fisher's Exact test (2 age-appropriate/below-age-appropriate x 3 viable strategy types) was run. Fisher's Exact Test was used because it identifies significant differences between groups of nominal data when at least one group has five or fewer observations (Field, 2009). Significance was set at .05. Results indicated no significant differences between the two groups (p = .931). This indicates spoken English comprehension was not related to the types of viable strategies children used; the p-value being so close to 1.0 suggests spoken English comprehension had a minute difference, if any at all, with oral d/hh children's viable strategy use.

Figure 14

Viable Strategy Types by Spoken English Comprehension



Note. N = Codable strategies

Summary

Results showed that the oral d/hh children provided a correct answer 4% to 52% of the time, indicating a general correctness rate well below 50%. In terms of strategy-type use, oral d/hh children used modeling most frequently, counting some of the time, and fact-based strategies least often overall. However, within story problem types, only six of the nine one-step arithmetic story problems were solved most often using a viable modeling strategy. The remaining three were solved using viable counting strategies most often, or either an equal amount of viable modeling and counting or viable modeling and fact-based strategies. The next chapter places these results within the context of the greater literature, particularly related to signing d/hh and hearing children's one-step story problem-solving. These considerations then develop conclusions and recommendations for teachers and researchers regarding oral d/hh children's one-step story problem-solving.

CHAPTER V

DISCUSSION

Introduction

Few studies in the past have highlighted d/hh children's story problem-solving (e.g., Edwards et al., 2013; Hyde et al., 2003; Nunes & Moreno, 1998; Frostad & Ahlberg, 1999). Two studies in particular have examined the story problem-solving of signing d/hh children, finding that their strategy use and relative story problem difficulty were both similar and different to that of hearing children; ultimately, it was suggested that signing d/hh children may progress in their story problem-solving along a different trajectory than hearing children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). However, no known studies have examined whether oral d/hh children's story problem-solving is more similar to signing d/hh children, hearing children, or is instead distinct. The present study observed oral d/hh children solving nine one-step arithmetic story problems drawn from past parallel research with signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). Identifiable strategies were coded by general type as either a modeling (to include modeling and direct modeling), counting, or fact-based strategy and then as viable or non-viable to determine oral d/hh children's relative problem difficulty. Descriptive and non-parametric statistical tests analyzed the patterns, allowing conclusions to be drawn about oral d/hh children's story problem-solving, and answering the following research questions:

1. What patterns of viable and non-viable strategies are used by d/hh children, kindergarten through Grade 3, who use listening/spoken English as their primary communication?
2. What is the relative difficulty of one-step arithmetic story problems for d/hh children, kindergarten through Grade 3, who use listening/spoken English as their primary communication?
3. How do patterns of strategy use and relative story problem difficulty compare between d/hh children, kindergarten through Grade 3, who use listening/spoken English as their primary communication and have age-appropriate English comprehension or higher, and those who have below-age-appropriate English listening comprehension?

Comparing Three Populations

Results from the present study reveal that oral d/hh children's story problem-solving to be a complex process, neither completely comparable to past research with hearing children (Carpenter et al., 2015) nor with signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). In the present study oral d/hh children exhibited patterns similar to signing d/hh children, (i.e., relative story problem difficulty) and hearing children (i.e., percentage of viable strategy use), as well as their own distinct patterns (i.e., viable strategies typically leading to an incorrect answer).

Generally, it can be concluded that oral d/hh children's one-step arithmetic story problem-solving is slightly more similar to their signing d/hh than hearing peers. However, Ansell and Pagliaro's (2006) conclusion that signing d/hh children develop along their own trajectory cannot be unequivocally applied to oral d/hh children as well because the parallel data supporting that conclusion were not nearly as clearly defined in the present study, as discussed below.

Patterns of Viable and Nonviable Strategy Type Use

All three groups of children—oral d/hh, signing d/hh, and hearing—used modeling, counting, and fact-based strategies to solve one-step arithmetic story problems (Carpenter et al., 2015; Pagliaro & Ansell, 2012).

Between Story Problem Types

Overall, the oral d/hh children used one strategy type (modeling) to solve eight of the nine story problems, and they used a counting strategy on the single easiest story problem (JRU), signing d/hh children also used one strategy type to solve all nine of the story problems (Pagliaro & Ansell, 2012), although a different strategy (counting). Hearing children, on the other hand, often use more than one strategy, counting or fact-based strategies (i.e., more abstract) on an easier story problem and then switching to modeling or counting (i.e., less abstract) on a more difficult story problem (Carpenter et al., 1999, 2015). In this way, oral d/hh children's pattern of strategy type use between story problem types was more similar to their signing d/hh peers in that both groups predominantly used a single strategy type to solve most if not all of the story problems.

However, when considering the types of *viable* strategies children use to solve story problems, a different pattern emerges. Here, it is not certain based on the present data whether oral d/hh children, like their signing d/hh and hearing peers, elect to use more concrete viable strategy types on relatively more difficult story problems. Oral d/hh children's viable modeling and counting strategy use was not consistent across story problem types. They predominantly used a viable counting strategy to solve both the easiest (JRU) and seventh (PU) most difficult story problem type, an equal amount of viable counting and modeling strategies on the third-easiest story problem type (JCU), and predominantly used a viable modeling strategy on the remaining six story problems. Signing d/hh children most frequently used a viable counting strategy to solve the two easiest story problems and used a viable modeling strategy most frequently to solve the remaining seven (more difficult) story problems (Pagliaro & Ansell, 2012). From this, the researchers suggested signing d/hh children's shift to more viable modeling strategies on more difficult story problems indicated they recognized this difficulty and, as their hearing peers do when using viable or non-viable strategies, switched to a more concrete strategy (Pagliaro & Ansell, 2012). Given that the present data regarding oral d/hh children's viable strategy use across story problem types does not exhibit the same certain patterns, it cannot yet be conclusively stated whether oral d/hh children are similar to their signing d/hh and hearing peers in regards to viable strategy use. The present data—wherein oral d/hh children used more abstract counting strategies on two of the relatively easier story problem types—suggests the possibility of strategy-type switching, but further research is needed to reach a more definitive conclusion especially given the small number of oral d/hh students who had viable strategies on the relatively more difficult problems.

Differences in Predominant and Viable Strategy Use

Although oral d/hh children elected to use a modeling strategy most often overall, they used viable counting strategies (65%) more often than viable modeling (62%) or fact-based strategies (50%). This indicates that while they chose counting strategies less frequently, they were more likely to use one appropriately. This is reflected in the predominant use of a viable modeling strategy to solve eight of the nine story problems, with JRU being the exception. An inverse relationship was seen with signing d/hh children who used counting strategies most often but were more likely to use a viable modeling strategy (Pagliaro & Ansell, 2012). In studies by Carpenter and colleagues (Carpenter & Moser, 1984; Carpenter et al. 1993), results of hearing children K-3 varied (depending on child, problem, and quantities), although a study by Carpenter et al. (1993) of only kindergarten hearing children did show that the kindergarteners used the same predominant strategy and viable strategy most often (Carpenter et al., 1993).

The findings related to d/hh children perhaps indicate an effect of instruction including different amount of time spent on story problem-solving in school. The hearing were educated by teachers who were training in and were implementing CGI-based instruction in their classrooms where instruction is based on children's thinking. They may have had more opportunities than the d/hh children to work with story problems and discuss their strategies (Carpenter et al., 1989; Fennema et al., 1992). In contrast, survey studies with teachers of the deaf suggest they do not typically provide daily story problem practice (Kelly et al., 2003; Pagliaro, 1998; Pagliaro & Ansell, 2002). That hearing children simply experience story problem-solving more often than d/hh may account for this distinction between d/hh and hearing children's viable strategy use. At the very least this suggestion underscores the importance of d/hh children working regularly, if not daily, on story problems.

Additionally, the CGI program that significantly increased hearing children's story problem-solving included frequent discussions where children described, defended, and challenged the different strategies they chose to use (Carpenter et al., 1989; Fennema et al., 1992). Further highlighting the importance of such discussion, a teacher using the CGI program in a classroom that included (hearing) children with special educational needs stated she had not realized the improvement that simply talking about story problem-solving would bring to all her students' work (Moscardini, 2014). Thus, simply providing story problems to d/hh children may not be enough to address this disconnect between predominant and viable strategy use. Having conversations *about* story problem-solving strategies may be a critical factor as well.

Viable Strategy Use and Correctness

Analyses of how often children use a viable strategy have been conducted using data from six story problems (JCU, SRU, PU, WU, JCU, and CDU) with oral d/hh (k – Gr. 3), signing d/hh (k – Gr. 3), and hearing (kindergarten) children. Six instead of nine story problems were used in this analysis to parallel similar analyses conducted with hearing (Carpenter & Moser, 1984) and signing d/hh (Ansell & Pagliaro, 2006) children. Of the 80 codable strategies oral d/hh children used on these six story problems, 75% (N = 60) were viable. Signing d/hh children used a viable strategy 54% (N = 189) of the time (Ansell & Pagliaro, 2006). Although Carpenter et al. (1993) documented hearing kindergarteners using a viable strategy 73% of the time, comparable data for hearing children in Grades 1-3 were not found and therefore a true comparison to oral d/hh and signing d/hh cannot be made. Although it is not known if these percentages are significant, they do suggest that further research designed to analyze the percentage of viable strategy use between groups is needed.

While on the surface oral d/hh children with below-age-appropriate spoken English comprehension used more viable strategies than their age-appropriate oral d/hh peers, when correctness is factored in, a different pattern emerges. Here, oral d/hh children with age-appropriate spoken English comprehension used a viable strategy and then arrived at the correct answer significantly more often (p = .006) than below-age-appropriate oral d/hh children. Why below-age-appropriate oral d/hh children arrived at significantly fewer correct answers after using a viable strategy may be linked to previous research documenting how children comprehend different types of stories based on counting and/or computation skills.

Viable strategies accurately represent what is described in a story problem (Carpenter et al., 2015). To solve a story problem, children must first create a representation of the known quantities and their relationship (i.e., engage a viable strategy) and then apply arithmetic or counting to that representation to find the unknown quantity (i.e., carry out a final step to arrive at the correct answer; Kintsch & Greeno, 1985). The above findings indicate below-age-appropriate oral d/hh children were just as successful at representing what the story problems described as their age-appropriate oral d/hh peers, but they struggled significantly to subsequently apply counting or arithmetic to their representations. While it is known that children can struggle to use a viable strategy if the quantities are too large (Carpenter et al., 2015), three counting and number tasks (drawn from Pagliaro & Ansell, 2012) were used in the present study to ensure children were not given story problems with quantities that were too large. Thus, there may be an issue in the expressive counting skills or calculation skills of non-age-appropriate oral d/hh children.

Edwards et al. (2013) investigated links between oral d/hh children's spoken vocabulary comprehension and arithmetic computation. They found that the oral d/hh children's spoken vocabulary comprehension scores were significantly ($p < .001$) correlated with their arithmetic computation and counting scores Oral d/hh children's vocabulary scores ($p < .001$) and counting/arithmetic scores ($p < .01$) were both significantly lower than the hearing participants' (Edwards et al., 2013). Similar results were found in studies of hearing children. In general education, studies show hearing children's spoken language comprehension scores are significantly related to their arithmetic computation and counting accuracy, wherein the higher one's language score the higher one's accuracy. This finding has been reported for hearing children with specific language impairments (including phonology, morphology, syntax; Cowan et al., 2005; Donlan et al., 2007), dual language learners (Méndez et al., 2019), learning disabilities (Cowan & Powell, 2014; Fuchs et al., 2006), and hearing children from low-income backgrounds (Gjicali et al., 2019). That these same findings were identified in such different contexts speaks to the powerful impact spoken language comprehension can have on children's mathematical counting and arithmetic computation.

Given this body of literature, the below-age-appropriate oral d/hh children in this study may have struggled to arrive at the correct answer due to inaccurately carrying out counting or arithmetic computation linked to their language difficulties. This conclusion reinforces the importance of oral d/hh children's language comprehension upon multiple aspects of their mathematics including both specific skills such as counting and arithmetic computation as well as more complex processes such as one-step arithmetic story problem-solving. Also revealed through this finding is that oral d/hh children's spoken language comprehension may be critical to their one-step story problem-solving not only for initial access and comprehension as surmised at the outset of this study, but for *all* steps in the story problem-solving process.

Relative Story Problem Difficulty

Relative story problem difficulty is defined by the percentage of viable strategies used to solve a particular story problem type; story problems solved with a higher percentage of viable strategies are considered easier than story problems solved with a lower percentage of viable strategies. The formal analysis of signing d/hh children's relative story problem difficulty did not include the MULT, PD, or MD story problems (Ansell & Pagliaro, 2006), although their full pattern of relative difficulty is published in a paper describing story problem-solving strategy use (Pagliaro & Ansell, 2012). As such, the following comparisons only included six story problem types: JRU, SRU, PU, WU, JCU, and CDU. Additionally, to present a truer comparison, only oral d/hh children with age-appropriate spoken English comprehension were included so that both samples present children with age appropriate language comprehension.

Overall, as shown in Table 5, the patterns of relative story problem difficulty between oral d/hh and signing d/hh children were almost identical.

Table 5

Patterns of Relative Story Problem Difficulty Between DHH Populations

Language	**Story Problem Types (Easiest to Difficult)**					
Oral d/hh age-appropriate	JRU	WU	JCU	SRU	PU	CDU
Signing d/hh*	JRU	WU	JCU	PU	SRU	CDU

Note. * Ansell and Pagliaro (2006)

Signing d/hh children found the JRU and WU story problem types significantly easier than all other types of story problems (Ansell & Pagliaro, 2006). Oral d/hh children also found the JRU and WU story problem types the easiest although no statistical analyses were conducted to determine significance. In their conclusion, Ansell and Pagliaro (2006) describing why JRU and WU were significantly easier than other story problems stated what made a story problem relatively easy for the signing d/hh children was whether the story problem's solution required sums of sets or not. This conclusion coupled with the present findings may indicate that d/hh children, regardless of language, find a one-step arithmetic story problem relatively easy if it requires the sums of sets (JRU, WU). While more research is needed to extend and clarify d/hh children's relative story problem difficulty, this finding may too reflect the mathematics instruction they receive.

Multiple sources in deaf education suggest mathematics vocabulary, particularly within story problems, can be particularly difficult for d/hh students to comprehend (Barham & Bishop, 1991; Hyde et al., 2003; Kidd, 1991; Kidd et al., 1993; Serrano Pau, 1995; Zevenbergen et al., 2001). Additionally, other sources recommend that d/hh students receive direct instruction in mathematics vocabulary (Barbosa, 2014; Easterbrooks & Stephenson, 2006). Collectively, these recommendations distinctly focus on developing mathematics vocabulary but do not concurrently discuss developing a "big picture" understanding of what is happening within a story problem. If this focus within the literature has extended to teaching, the critical dimension of d/hh children's story problem-solving may stem from vocabulary traditionally understood to describe a sum.

General education teachers (i.e., non CGI) sometimes directly instruct their students to use the "key word" strategy—the process of using words in the story problem to select a strategy (Jonassen, 2000; Powell & Fuchs, 2018; Sowder & Schappelle, 1995). For example, when a child sees "more" they are told to add. This is problematic because research with hearing children shows those who use this key word strategy do not understand the "big picture" within a story problem, and therefore struggle when approaching more complex and unfamiliar story problems (Cummins et al., 1988; Hegarty et al., 1995; Wiest, 2003), or story problems where the key word does not match the operation to be done with the given order of numbers (e.g., the JCU problem in the present study). Children using a key word story problem-solving approach are considered less proficient story problem solvers. Given the focus on vocabulary within mathematics instruction in deaf education as suggested above, it is possible that the oral d/hh children in the present study were correctly or incorrectly attending to the key words they heard in the problems, resulting in the JRU (key word *altogether*) story problem being the easiest. Thus, the critical dimension of relative story problem difficulty for d/hh children may lie within factors related to the mathematics instruction they receive.

A non-statistical review of all nine story problems found the same story problem types the most difficult for oral d/hh and signing d/hh. Oral d/hh found the PD and MD story problems among the most difficult. These require a child to divide one known quantity into groups and then either count the number of groups (MD) or the number within one group (PD). These division story problems were eighth and nineth most difficult for the oral d/hh children and the seventh- and eighth-most difficult story problem types for signing d/hh children, with the Compare problem (CDU) being the most difficult (Pagliaro & Ansell, 2012). Thus, the d/hh children, regardless of language preference, appear to find the traditional division story problem types relatively more difficult than most, if not all, other story problem types.

PD and MD story problem types require that a child understands that groups of objects can be counted just as individual objects can, and that a larger amount can be composed of multiple, equal, smaller amounts (Carpenter et al., 2015). This understanding is considered advanced because it moves beyond representing and counting individual things (i.e., dots, fingers, cubes, etc.) to representing and counting groups of things as one. Given the results from the present study and that of Pagliaro and Ansell (2012) it may be that- d/hh children are not as proficient or educated at this point in moving from modeling and counting individual things to modeling and counting groups of things as one. In addition, the ability to *divide* an entire group into multiple equal smaller sets may be a critical distinction for oral d/hh children in the present study, as signing d/hh children found the multiplication story problem type—where smaller equal quantities are *grouped* into a larger set—quite a bit easier than did the oral d/hh children (Pagliaro & Ansell, 2012). Thus, two important skills may be a focus for future work with d/hh children's problem-solving: (1) the understanding of modeling and counting individual things to modeling and counting groups of things as one, and (2) the distinct skills of composing and decomposing sets of multiple equal groups.

The Non-Significance of Spoken English Comprehension

There is a long and well-established body of literature documenting that higher *reading* comprehension leads to significantly higher rates of story problem correctness for oral d/hh children based on written story problems (Frostad & Ahlberg, 1999; Hyde et al., 2003; Kelly & Mousley, 2001; Kidd, 1991; Kidd & Lamb, 1993; Serrano Pau, 1995; Swanwick et al., 2005; Zevenbergen et al., 2001). However, reading comprehension is not the same as spoken language comprehension because each requires a different skill set, comprehending language in a distinct mode. Thus, listening comprehension as a story problem-solving variable was considered in the present study. The following section summarizes results of multiple non-parametric statistical analyses testing for significant differences between spoken English comprehension levels upon a variety of oral d/hh children's story problem-solving factors. Table 6 outlines these findings.

Table 6

Summarizing Significant Differences Across Multiple Story Problem-Solving Factors

Variable	**Significant Difference (*p* < .005)**
Correctness	No
Viable/Non-Viable Strategy Use	No
Strategy Type Use	No
Viable Strategy Type Use	No
Viable Strategy Type Use	No
Viable Strategies Leading to Correct Answer	Yes

There were no significant differences in oral d/hh children's story problem-solving based on spoken English comprehension, except for how often a correct answer was given after a viable strategy was used (discussed earlier). Why spoken English comprehension was so insignificant is uncertain. D/HH children with higher ASL comprehension were significantly more proficient story problem-solvers than d/hh children with lower ASL comprehension (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). A plethora of research has concluded that lower levels of language comprehension are at least partially responsible for d/hh children's lower mathematics outcomes (e.g., Antia et al., 2009; Daniele, 1993; Edwards et al., 2013; Marschark et al., 2011; Qi & Mitchell, 2011; Swanwick et al., 2005; Traxler, 2000), including story problem-solving (Barbosa, 2014; Blatto-Vallee et al., 2007; Kelly & Mousley, 2001; Kidd, 1991; Serrano Pau, 1995; Zevenbergen et al., 2001). Given these indicators, it was expected that spoken English comprehension would be a significant factor for multiple aspects of oral d/hh children's story problem-solving. That this was not the case is the most surprising finding of this study. This is currently the only known study to examine oral d/hh children's spoken English comprehension in relation to their story problem-solving. Multiple influences could underlie this surprising result, although further research is needed to advance and confirm these possibilities. Some possible factors are related to the spoken English language screening measure, while others are related to limitations of the present study.

Related to the spoken English comprehension measure, of the nine oral d/hh children with below-age-appropriate spoken English comprehension, only one had a score well below the CELF-5 (Wiig et al., 2013) cut-off while the remaining eight had scores just below the cut-off. That is, age-appropriate oral d/hh children had a scaled score of 7 or higher (Wiig et al., 2013); of the below-age-appropriate oral d/hh children, eight had a scaled score of six and one had a scaled score of two. If the cut-off (6/7) represents a minimally acceptable level of language, it is possible spoken English comprehension was not significant for oral d/hh children's story problem-solving because 96% (N = 23) of participants had the requisite command of English necessary to comprehend spoken one-step arithmetic story problems. Further research, potentially with oral d/hh children displaying a wider spread of below-age-appropriate language comprehension scores, is needed to clarify this possibility.

Three procedural limitations also may have contributed to this finding of no difference based on language comprehension as well. First, only the *Sentence Comprehension* subtest was used (Wiig et al., 2013) to measure spoken English comprehension in order to strike a balance between a child's total missed class time and a full measure of the child's spoken English comprehension within the study's procedures. While the testing manual clearly states these subtests can be used as standalone measures, it also states that using the CELF-5 in its entirety is the most robust measure of language offered by this assessment (Wiig et al., 2013). Future research using more in-depth measures of spoken English comprehension is needed to explore this possibility.

Second, studies also show that hearing children's working memory skills significantly predict their one-step arithmetic story problem-solving accuracy (Andersson, 2007; Friso-Van den Bos et al., 2013; Zheng et al., 2011), suggesting that increased abilities to both hold and manipulate information in working memory, and directly recall and reproduce information, can contribute to more accurate story problem-solving. In order to reduce data collection time, the oral d/hh children's working memory was not measured in the present study, however, past research has found that oral d/hh children's working memory capacity to be less than hearing children's, related to issues of encoding wherein oral d/hh children must store degraded auditory information in their working memory (AuBuchon et al., 2015, 2019; Davidson et al., 2018; Nittrouer et al., 2017; Pisoni & Cleary, 2003). Future research exploring the interactions between oral d/hh children's one-step arithmetic story problem-solving and their working memory is needed to address this limitation.

Third, the relatively small sample size may have led to numerous Type I errors, where a significant difference truly exists but the statistical tests do not find it. Using non-parametric, instead of parametric, statistical tests attempted to account for the large variety in sample sizes between spoken English comprehension groups to reduce the possibility of Type I errors (Field, 2009). However, this possibility was only reduced, not eliminated. Additionally, non-parametric tests are not as sensitive as parametric tests (Field, 2009). Future studies with larger samples, particularly larger samples with a more equal number of participants in each spoken English language comprehension group, will address these statistical limitations.

In summary, language comprehension provides access to one-step story problems, and higher vocabulary comprehension scores (Edwards et al., 2013) and phonological processing scores (De Smedt et al., 2010; Krajewski & Schneider, 2009; Simmons et al., 2008) have been linked to higher mathematics scores. However, this significant relationship between language and mathematics did not emerge in the present study of oral d/hh children's one-step arithmetic story problem-solving at the general level, and only presented as a significant difference to correctness within the analysis of viable strategies. Thus, further investigations are needed clarify this surprising finding.

Limitations

In addition to the above stated limitations of the CELF-5 (Wiig et al., 2013), the present study is limited by several other factors. First, although the initial target sample size for this study was 54, the actual study included only 24 participants. While the planned ANOVAs were removed, the final sample size is still much smaller than anticipated. The small sample size also affected the confidence in any comparisons to hearing children (Carpenter et al., 2015) and signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012) both of which included greater sample sizes. These differences in sample size underscore the need for further research into d/hh children's story problem-solving, particularly oral d/hh.

Second, none of the participants in this study spent their entire school day integrated into a general education classroom. Past data indicates 86-88% of U.S. d/hh children spend part of their day in such a setting, and 61-64% spent most or all of their day there (Snyder et al., 2016, 2018). Thus, while the present data reflects some portion of the 20% of d/hh children who did *not* spend their day in a general education classroom (Snyder et al., 2016, 2018), these findings are not necessarily reflective of oral d/hh children as a whole subpopulation. Research is needed with oral d/hh who spend most, if not all, of their day in a general classroom. Doing so will ensure findings and conclusions regarding one-step arithmetic story problem-solving more accurately reflect the entire subpopulation.

Finally, hearing children's story problem-solving was defined using studies with children educated in CGI-using classrooms, a program known to significantly increase classroom teachers' use of story problems (e.g., Carpenter & Fennema, 1992; Carpenter et al., 1993; Carpenter et al., 1996; Franke et al., 2001; Franke & Kazemi, 2001; Peterson et al., 1989), and by extension, hearing children's story problem-solving correctness and flexible strategy use (Carpenter et al., 1989; Peterson et al., 1989; Villaseñor & Kepner, 1993). As it was not confirmed if any of the oral d/hh participants in this study were also educated in a CGI-based classroom, comparisons between oral d/hh with age-appropriate spoken English comprehension and hearing children are not necessarily direct in terms of classroom programming. This was also the case with the studies with signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012). Despite this potential educational difference between hearing and d/hh children, the comparisons were still made to preserve the parallel nature of the study wherein understandings of hearing (Carpenter et al., 2015), signing d/hh (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012), and now oral d/hh children's one-step arithmetic story problem-solving all draw upon an identical framework.

Recommendations and Conclusions

Recommendations for Teaching

Findings from this study inform several recommendations for teaching one-step story problem-solving to oral d/hh children. First, oral d/hh children need regular, if not daily, opportunities to solve one-step arithmetic story problems. Additionally, it is important that this exposure include *discussions* of story problem-solving, exposing children to a variety of strategies and story problem types. To foster this discussion, teachers may wish to begin with the relatively easiest story problem types, which require the sum of sets. Also, capitalize on oral d/hh children's propensity to use a modeling strategy by drawing pictures, acting things out, and providing manipulatives that represent the objects described within the story. In addition, teachers should model the language used to participate in such discussions for students to independently express their thinking and reasoning. Modeling could include using graphic organizers to represent one's thoughts and linking vocabulary to ideas represented in the story of the problem situation, not to an operation.

Finally, it is essential that teachers consider their student's spoken language comprehension skills when observing and assessing spoken one-step arithmetic story problems. Such consideration is critical because oral d/hh children's spoken language comprehension difficulties can significantly impact their brain-level language processing (Leybaert, 2003; Pénicaud et al., 2013; Skotara, 2012) and spoken language comprehension levels (Edwards et al., 2011; Geers et al., 2009; Geers & Sedey, 2012). Additionally, language comprehension struggles can impact children's counting/ computation accuracy, a finding identified with both oral d/hh (Edwards et al., 2013) and hearing children (Cowan & Powell, 2014; Cowan et al., 2005; Donlan et al., 2007; Fuchs et al., 2006). Both language comprehension and the ability to count/compute are necessary to solve one-step arithmetic story problems. The present study illustrated this assertion, finding that oral d/hh children's spoken language comprehension was significantly related to their correctness when using a viable strategy. Thus, considering oral d/hh children's language comprehension during observation and assessment is critical.

Recommendations for Research

Further research into oral d/hh children's story problem-solving is needed to address the limitations of the present study. Studies with larger sample sizes, conducted with oral d/hh children who attend an integrated setting full-time are needed to overcome the sample limitations of the present study. Similarly, studies with oral d/hh children whose spoken English comprehension is clearly below/above the language comprehension cut-off is also needed in order to establish a clearer, more definitive indication of the impact of spoken English comprehension on oral d/hh children's story problem-solving. Related to the question of language comprehension is the question of auditory working memory, and whether an oral d/hh child's one-step arithmetic story problem-solving and their working memory, that is, their ability to effectively recall all parts of the story problem, are somehow related.

Additionally, given that oral d/hh children's story problem-solving neither precisely fits within patterns of signing d/hh children's nor hearing children's story problem-solving, but rather embodied elements of both in addition to unique patterns not previously identified, future research could refine the present understanding of oral d/hh children's story problem-solving by comparing oral d/hh, signing d/hh, and typical hearing (non-CGI) children's story problem-solving within the same study. Doing so will increase the validity of findings regarding similarities and differences between groups because the comparative analyses can be made at one controlled time.

These findings also highlight the need for studies into the story problem-solving of d/hh children who use other communication systems (i.e., Sign Supported Speech, Cued Speech), research not known to exist currently. Given the distinct differences uncovered between oral d/hh and signing d/hh children, these findings clearly cannot be assumed to apply to d/hh children who use these systems as well. Rather, the story problem-solving of d/hh children using other communication systems is best supported through research with those who communicate in the same way. All these recommended studies should use strategy type use, viable strategy type use, and correctness, to evaluate one-step arithmetic story problem-solving behavior. Doing so ensures the child's work is understood through multiple factors, greatly reducing the possibility of a child's story problem-solving skill being misrepresented or misunderstood.

Summary

This chapter placed results within the context of literature from both general and deaf education, particularly related to signing d/hh and hearing children's one-step arithmetic story problem-solving. Oral d/hh children's one-step arithmetic story-problem solving does not specifically mirror results from past parallel research with signing d/hh children (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012) nor hearing children (Carpenter et al., 2015); however, oral d/hh children's patterns of strategy type use, viable strategy use, correctness, and relative story problem difficulty mirrored their signing d/hh peers' results more often. The most surprising finding of this study was the limited role spoken English comprehension appeared to play in the oral d/hh children's one-step arithmetic story problem-solving. Spoken English comprehension considered within the contexts of working memory and links between language comprehension and computation/calculation accuracy revealed such language comprehension is critical for oral d/hh children to successfully complete all steps in solving a one-step arithmetic story problem from initial access/comprehension through to final counting/computation. Limitations of this study included factors related to language screening measures, sample size, sample representativeness, and the educational contexts of comparative populations, were also presented. To conclude this chapter, the conclusions and limitations supported multiple recommendations for both teachers and researchers.

This dissertation study set out to describe the one-step arithmetic story problem-solving of oral d/hh children in kindergarten through third grade. Specifically, this study sought to address the noted absence of this population within research on children's one-step story problem-solving, and to do so in such a way as to address concerns raised within deaf education regarding sample homogeneity. In providing an initial glimpse into oral d/hh children's one-step arithmetic story problem-solving, this study extends what deaf and general education know about children's one-step story problem-solving to a new population while also defining directions for future research opportunities.

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APPENDIX A

STORY PROBLEM TYPES

Story Problem Type Abbreviations, In Order of Presentation

Title	**Abbreviation**
Join Result Unknown	JRU
Separate Result Unknown	SRU
Part/Whole: Part Unknown	PU
Part/Whole: Whole Unknown	WU
Compare Difference Unknown	CDU
Join Change Unknown	JCU
Multiplication	MULT
Partitive Division	PD
Measurement Division	MD

APPENDIX B

CODEBOOK

The codebook was extracted from IBM SPSS Statistics (Version 26). The descriptors define each variable and the labels define each code. Each row consisted of one child's codes, entered under the variable column headings.

Variable	Descriptor	Code	Label
Language	Child's CELF-5 score	0	Below-Age Appropriate
JRU_acc	code		
		1	Age Appropriate
	JRU answer correct?	0	No
		1	Yes
JRU_strat	JRU strategy type	1	Modeling
		2	Counting
JRU_acc		3	Facts
JRU_viable	JRU strategy viable?	0	Non-Viable
JRU_strat		1	Viable
SRU_acc	SRU answer correct?	0	No
		1	Yes
SRU_strat	SRU strategy type	1	Modeling
		2	Counting
SRU_acc		3	Facts
SRU_viable	SRU strategy viable?	0	Non-Viable
SRU_strat		1	Viable
PU_acc	PU answer correct?	0	No
		1	Yes
PU_strat	PU strategy type	1	Modeling
		2	Counting
PU_acc		3	Facts
PU_viable	PU strategy viable?	0	Non-Viable
PU_strat		1	Viable
WU_acc	WU answer correct?	0	No
		1	Yes

Variable	Descriptor	Code	Label
WU_strat	WU strategy type	1	Modeling
		2	Counting
WU_acc		3	Facts
WU_viable	WU strategy viable?	0	Non-viable
WU_strat		1	Viable
CDU_acc	CDU answer correct?	0	No
		1	Yes
CDU_strat	CDU strategy type	1	Modeling
		2	Counting
CDU_acc		3	Facts
CDU_viable	CDU strategy viable?	0	Non-Viable
CDU_strat		1	Viable
JCU_acc	JCU answer correct?	0	No
		1	Yes
JCU_strat	JCU strategy type	1	Modeling
		2	Counting
JCU_acc		3	Facts
JCU_viable	JCU strategy viable?	0	Non-Viable
JCU_strat		1	Viable
MULT_acc	MULT answer correct?	0	No
		1	Yes
MULT_strat	MULT strategy type	1	Modeling
		2	Counting
MULT_acc		3	Facts
MULT_viable	MULT strategy viable?	0	Non-Viable
MULT_strat		1	Viable
PD_acc	PD answer correct?	0	No
		1	Yes
PD_strat	PD strategy type	1	Modeling
		2	Counting
PD_acc		3	Facts
PD_viable	PD strategy viable?	0	Non-Viable
PD_strat		1	Viable
MD_acc	MD answer correct?	0	No
		1	Yes

Variable	Descriptor	Code	Label
MD_strat	MD strategy type	1	Modeling
		2	Counting
MD_acc		3	Facts
MD_viable	MD strategy viable?	0	Non-Viable
MD_strat		1	Viable